ACQAO Regional Workshop 2011

Nonlinear Quantum Interferometry with Bose Condensed Atoms

Chaohong Lee

State Key Laboratory of Optoelectronic Materials and Technologies,
School of Physics and Engineering,
Sun Yat-Sen University, Guangzhou 510275, P. R. China

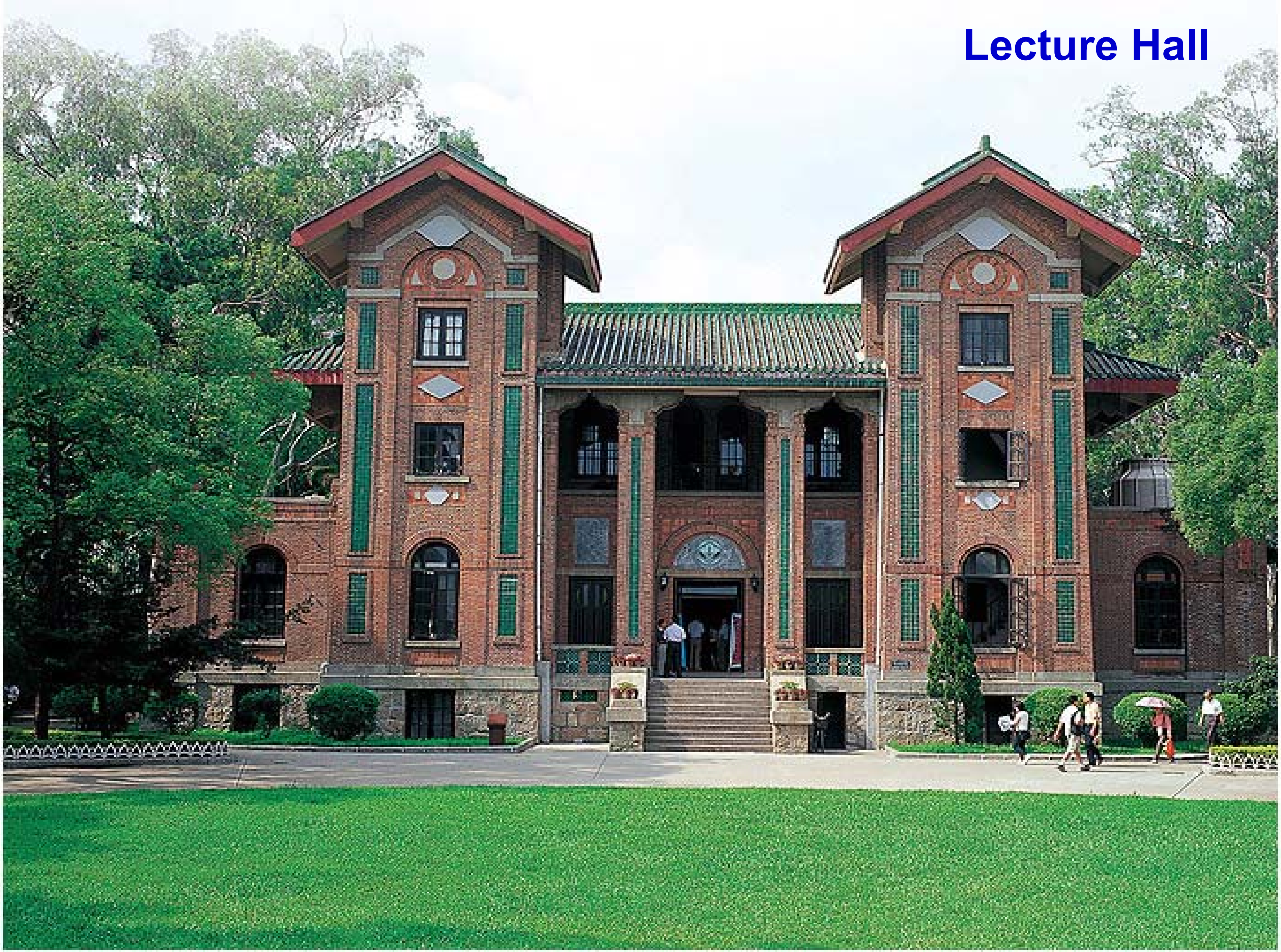
Email: lichaoh2@mail.sysu.edu.cn; chleecn@gmail.com
ResearcherID: www.researcherid.com/rid/A-1402-2008
Supported by: NNSFC, MOE, MOST

Where is SYSU?

Sun Yat-Sen

First revolutionist in modern China;
Establisher of R China


North Entrance
國立中山大學


Lecture Hall

Current research interests of my group

- Quantum interferometry (Bose-condensed atoms, ultracold trapped ions)
- Cavity-QED with Bose condensed atoms
- Non-equilibrium many-body quantum dynamics (TEBD, quantum spin model, Bose-Hubbard model, etc)
- Theoretical studies of quantum technology with ultracold atoms (high-precision measurements, quantum simulation, atom clocks, etc)

Outline

- **Introduction**
- Quantum metrology
- Interferometry with Bose condensed atoms
- **Matter-wave interferometry**
- Atomic matter-wave interference
- Nonlinear excitations
- Bose-Josephson junctions
- **Many-body quantum interferometry**
- Quantum spin squeezing and many-particle entanglement
- High-precision interferometry via spin squeezed state
- High-precision interferometry via NOON state
- **Summary and open problems**

1. Introduction

"Natural measures of quantity, such as fathoms, cubits, inches, taken from the proportion of the human body, were once in use with every nation," taught Adam Smith in his lecture "Money as the measure of value and medium of exchange," delivered in 1763. "But by a little observation," he continued, "they found that one man's arm was longer or shorter than another's, and that one was not to be compared with the other; and therefore wise men who attended to these things would endeavour to fix upon some more accurate measure, that equal quantities might be of equal values. Their method became absolutely necessary when people came to deal in many commodities, and in great quantities of them (*1*)." Smith's comments and the rationale underpinning them became increasingly urgent toward the end of the eighteenth century.

W. J. Ashworth, Metrology and the State: Science, Revenue, and Commerce, Science 306, 1314 (2004)

Measurement standards defined by human body (old-time UK people) or rice length (old-time Chinese people) are not accurate. They may change case by case.

1.1. Quantum Metrology

FUNDAMENTALS OF MEASUREMENT

SCIENCE VOL 306 19 NOVEMBER 2004

SPECIAL SECTION

Higher Standards

This special issue of *Science* looks at the development of precision measurement, how its tools have been developed and adapted for better performance, and how the standards used today may be further improved. Historically, measurements were often based on somewhat arbitrary local units. In his Viewpoint, Ashworth (p. 1314) describes, from a British perspective, the development of a standardized metrology as applied to weights and measures and how the burgeoning commerce of the industrial revolution drove its development.

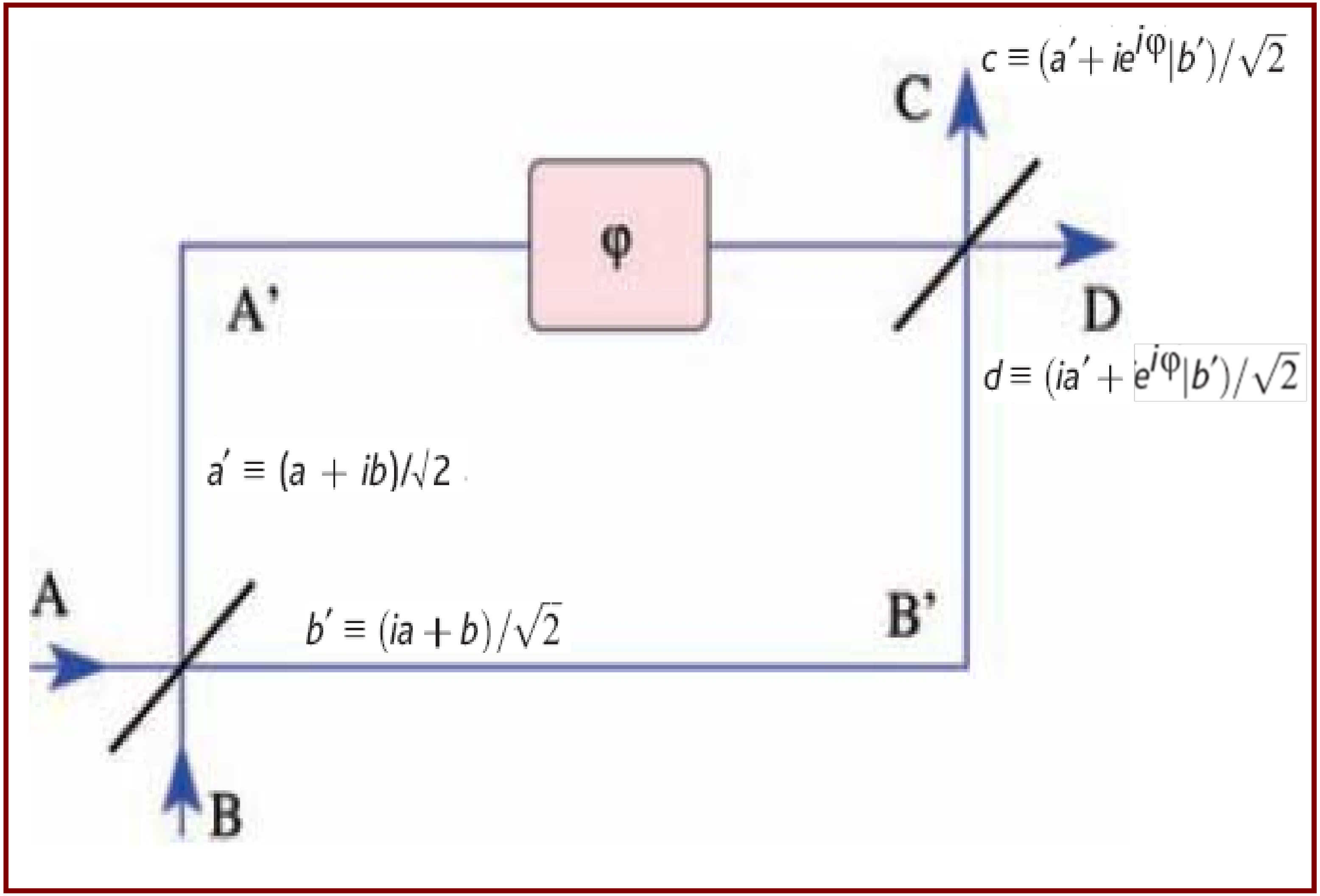
time
mass
length
...

Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

V. Giovannetti, S. Lloyd, and L. Maccone, Science **306**, 1330 (2004)

Quantum mechanics, through the Heisenberg uncertainty principle, imposes limits on the precision of measurement. Conventional measurement techniques typically fail to reach these limits. Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten using quantum strategies that employ "quantum tricks" such as squeezing and entanglement.

Mach-Zehnder interferometry



Ramsey interferometers

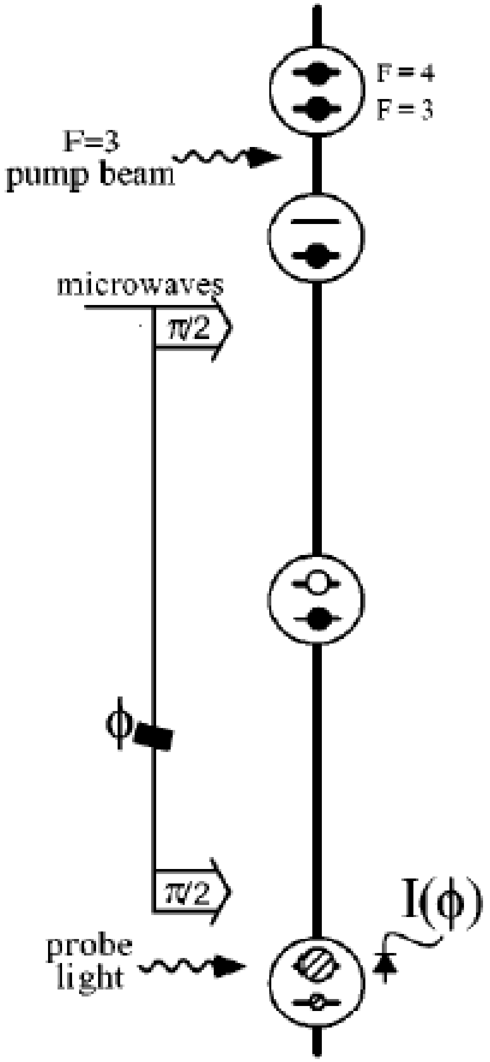
(1)prepare an initial state |1>;

(2)apply the first half-Pi pulse to create an equal superposition of |1> and |2>;

(3)accumulate a relative phase between |1> and |2> in the free evolution;

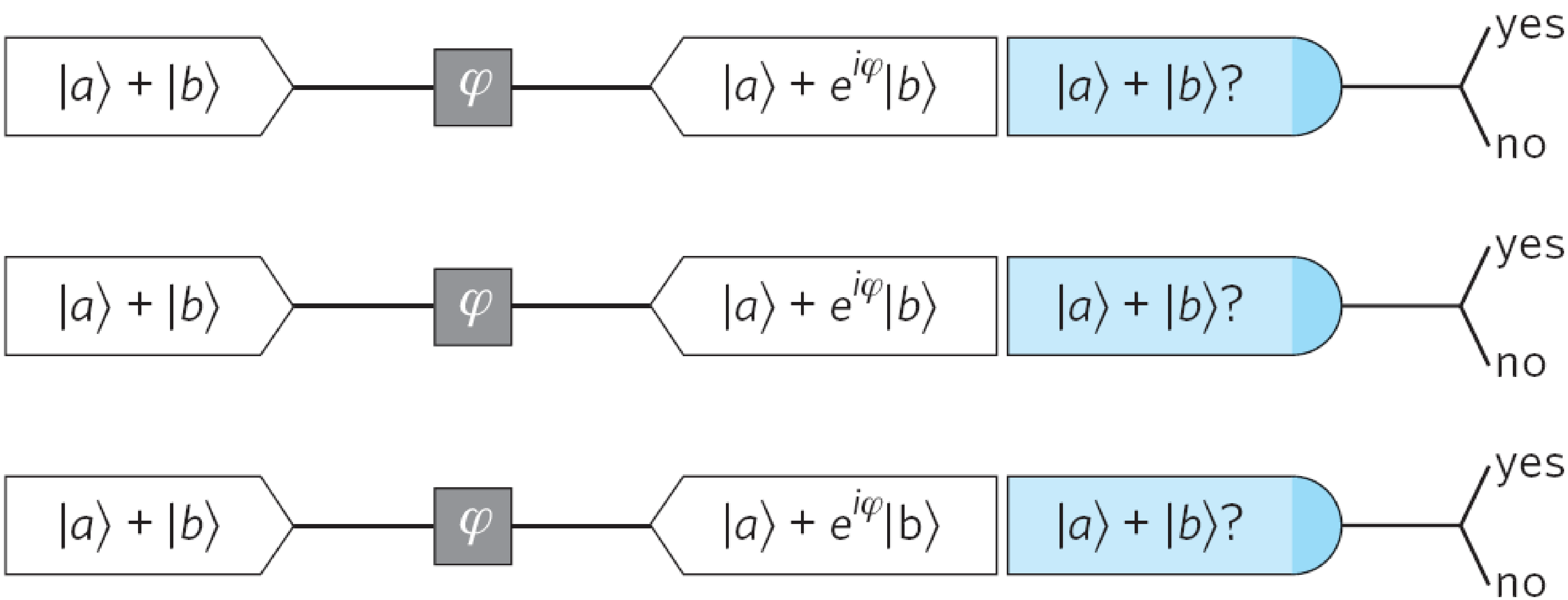
(4)recombine |1> and |2> via the second half-Pi pulse;

(5)detect the final state.



Atom Interferometry,
edited by P. Berman (Academic Press, San Diego, 1997)

Ramsey interferometry via independent particles
(general phase measurement)

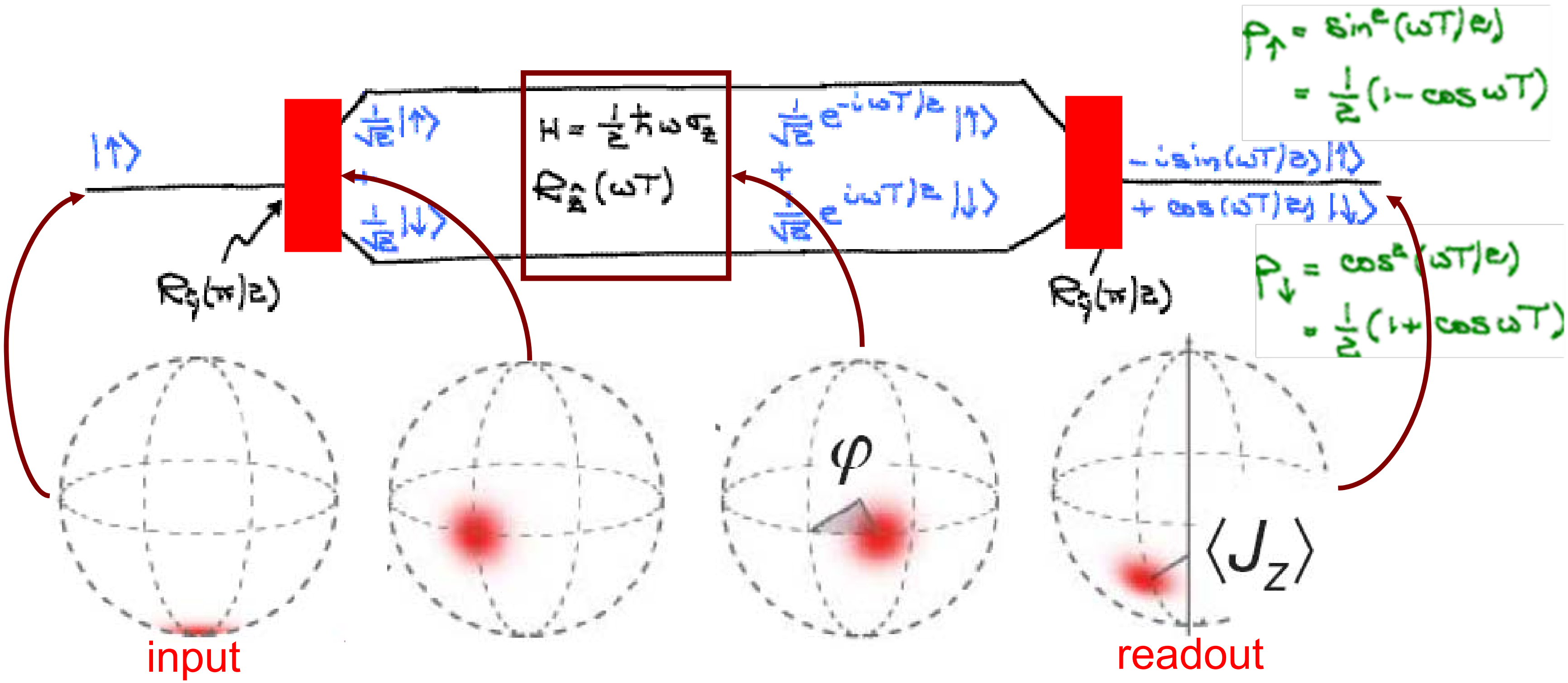


$$p = \frac{\#\text{yes (in } n \text{ repetitions)}}{n} \longrightarrow \frac{1 - \cos\varphi}{2}$$

Wineland et al., Spin squeezing and reduced quantum noise in spectroscopy. *Phys. Rev. A* ***46, R6797-R6800 (1992).***

Braunstein, Quantum limits on precision measurements of phase. *Phys. Rev. Lett.* ***69, 3598-3601 (1992).***

Frequency measurement via independent particles



***N* independent "particles"**

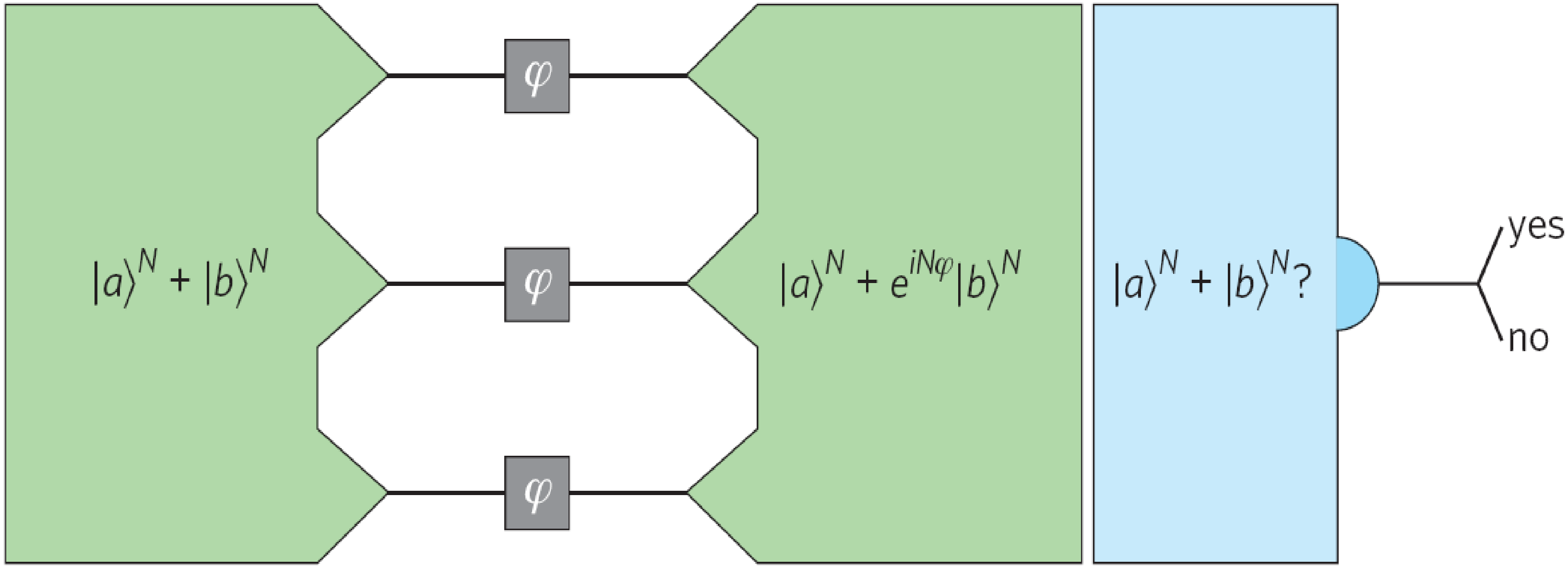
$$\text{(signal)} = \langle \sigma_z \rangle = -\cos \omega T$$

$$\text{(noise)} = \Delta\sigma_z = \sqrt{1 - \cos^2 \omega T} = |\sin \omega T|$$

$$\Delta(\omega T) = \frac{1}{\sqrt{N}} \frac{\text{(noise)}}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{N}}$$

standard quantum limit
(shot noise limit)

Ramsey interferometry via cat state (NOON state) (general phase measurement)

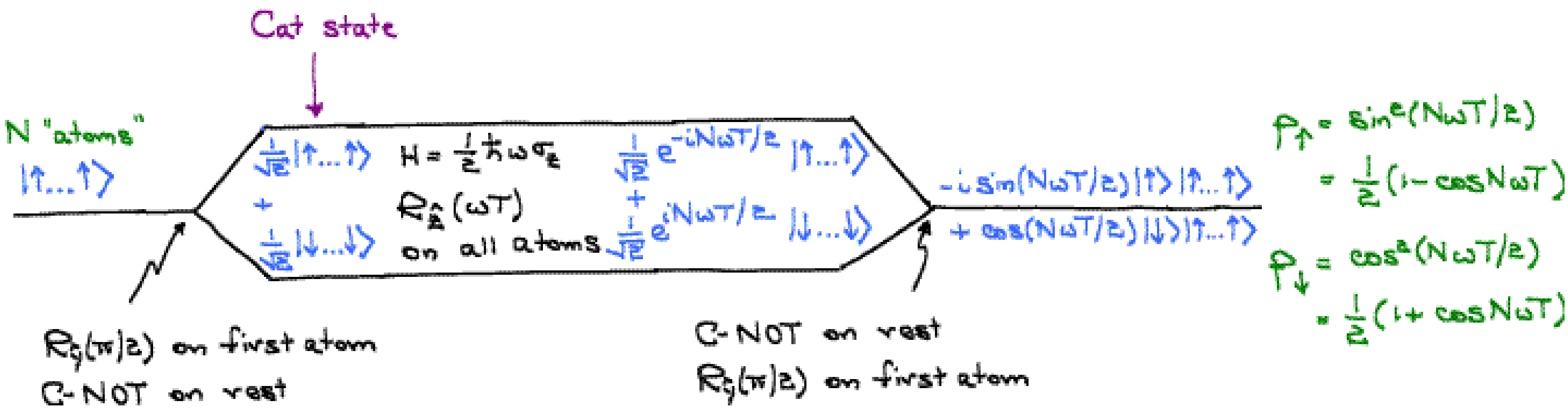


$$p_{\mathrm{ent}} = \frac{\#\text{yes (in } v \text{ repetitions)}}{v} \longrightarrow \frac{1 - \cos N\varphi}{2}$$

Wineland et al., Spin squeezing and reduced quantum noise in spectroscopy. *Phys. Rev. A* ***46, R6797-R6800 (1992).***

Braunstein, Quantum limits on precision measurements of phase. *Phys. Rev. Lett.* ***69, 3598-3601 (1992).***

Frequency measurement via cat state (NOON state)



- Cat-state preparation and read-out require distinguishing all atoms.
- How about indistinguishable systems? (BEC)

Fringe pattern with period 2π/*N*

$$\text{(signal)} = \langle\sigma_z\rangle = -\cos N\omega T$$

$$\text{(noise)} = \Delta\sigma_z = \sqrt{1-\cos^2 N\omega T} = |\sin N\omega T|$$

$$\Delta(\omega T) = \frac{1}{\sqrt{\nu}}\frac{\text{(noise)}}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{\nu}}\frac{1}{N}$$

Heisenberg limit

$\nu = \text{(number of trials)}$

***N* cat-state atoms**

1.2. Interferometry with Bose condensed atoms

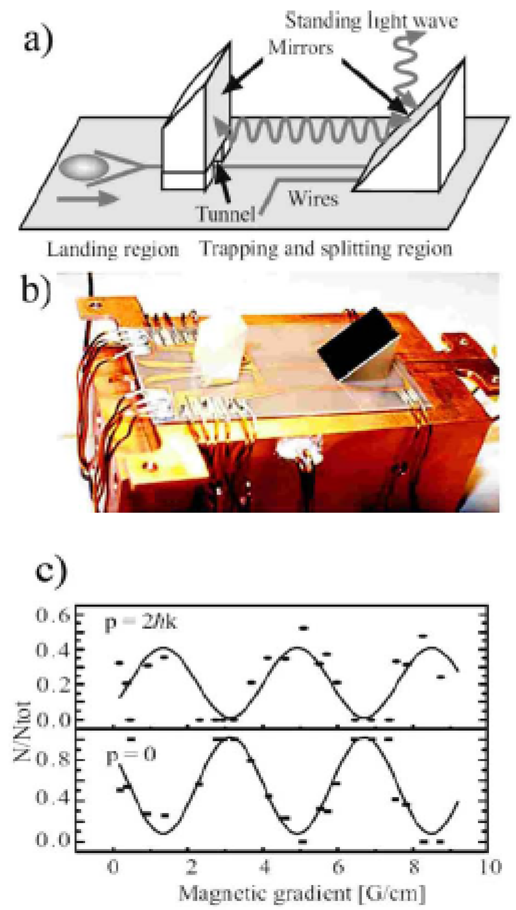
Michelson interferometers

(1) The BEC is split at t=0 into two momentum components $\pm 2kL$ using a double pulse of a standing light wave.

(2) A Bragg scattering pulse at t=T/2 then reverses the momentum of the atoms and the wave packets propagate back.

(3) At t=T the split wave packets overlap and a third recombining double pulse completes the interferometer.

To apply a phase shift between the two paths, a magnetic field gradient was turned on for a short time while the atom clouds were separated.



Wang et al., 2005, “An atom Michelson interferometer on a chip using a Bose-Einstein condensate,” Phys. Rev. Lett. **94, 090405.**

Double-well interferometers

(1) Preparation: a single BEC coherently splits into two by increasing the potential barrier.

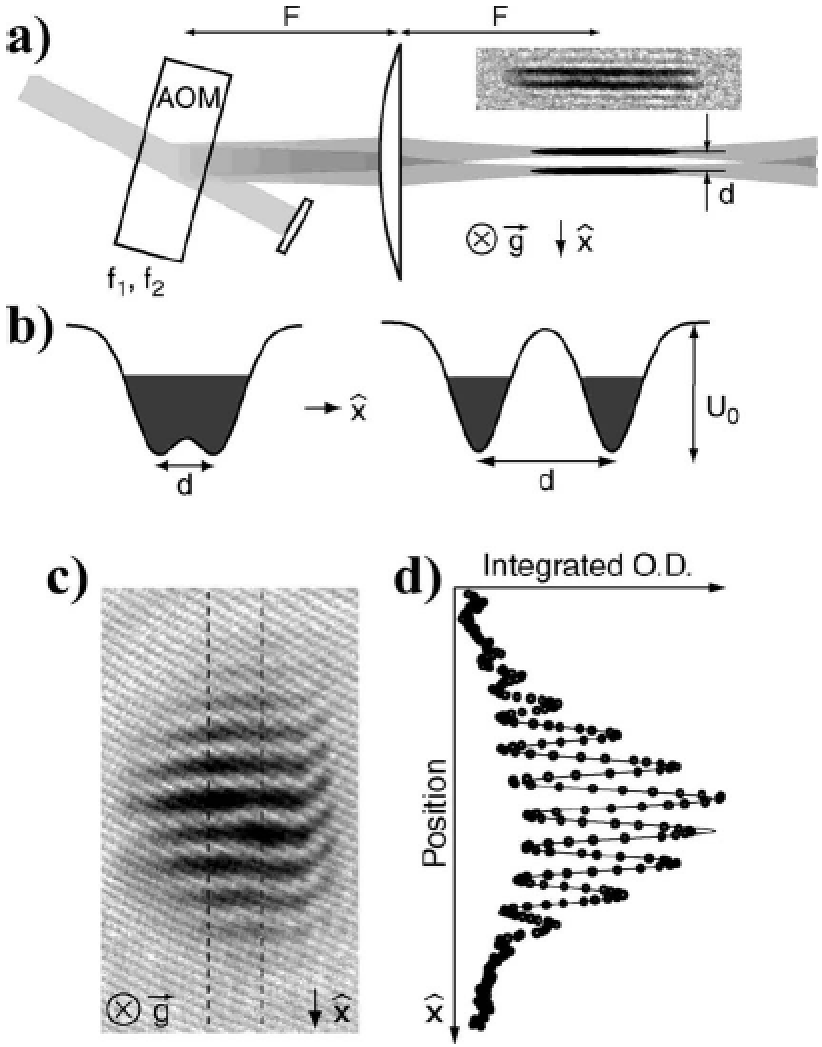
(2) Phase shift: an interaction may be used to induce the phase shift between two BECs.

(3) Interference: the split BECs in the two wells are recombined to observe the interference.

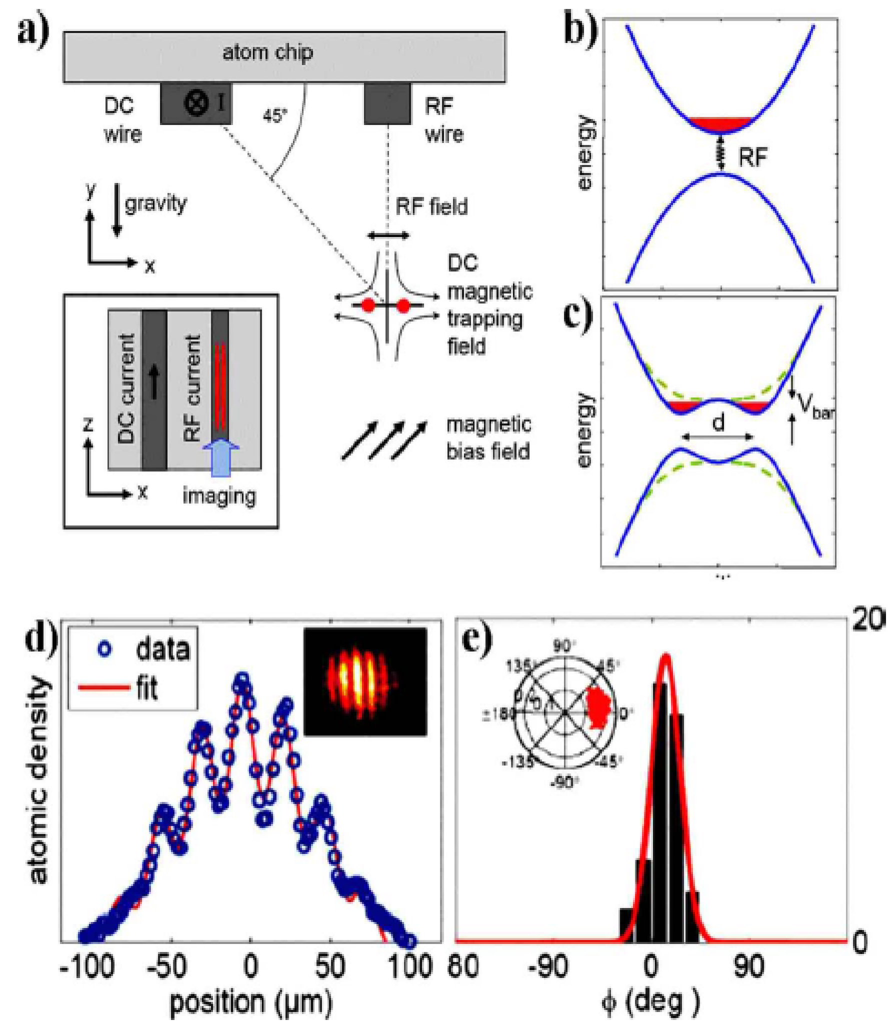
(I) Optical potentials (optical trap + laser barrier)
Shin et al., 2004, "Atom interferometry with Bose-Einstein condensates in a double-well potential," Phys. Rev. Lett. **92, 050405**.

(II) Magnetic potentials (atom chips)
Schumm et al, 2005, "Matter-wave interferometry in a double well on an atom chip," Nature Phys. **1, 57.**

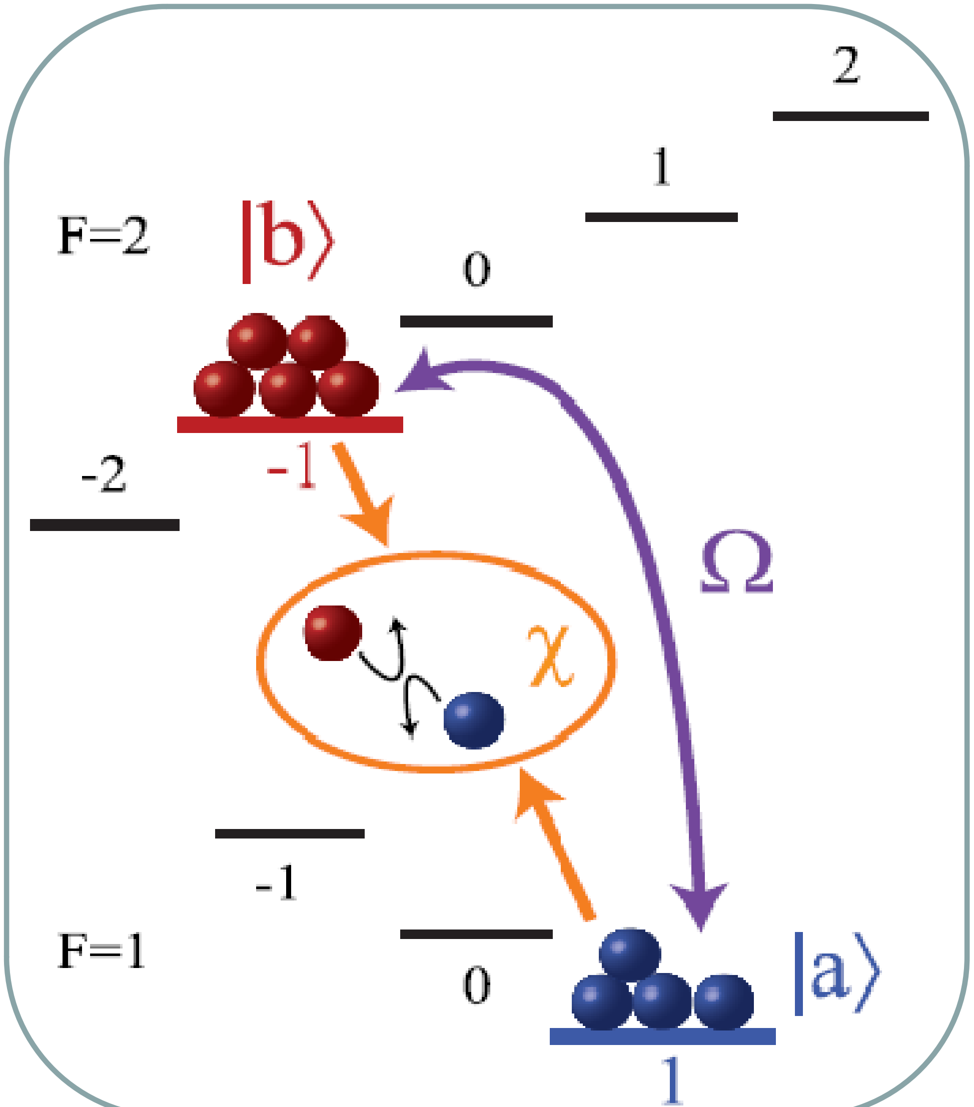
(I) Optical potentials
(optical trap + laser barrier)

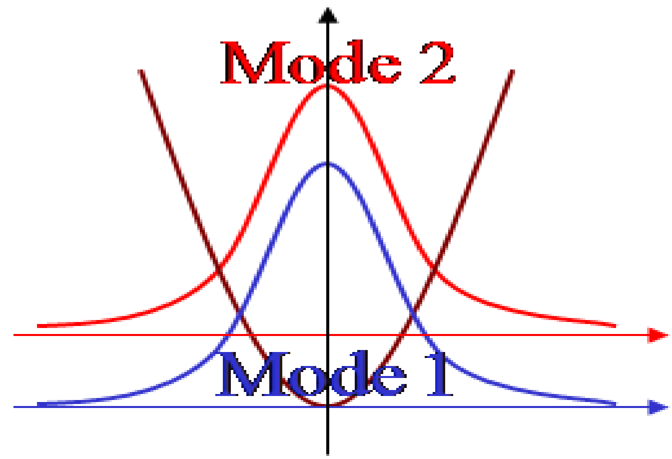


(II) Magnetic potentials
(atom chip)



Ramsey interferometers with two-component systems





JILA ,

LENS,

ANU,

Heidelberg Uni.,

SUT,

etc.

Potential Applications

(1) High-precision quantum frequency standards (atom clocks)

Atomic transitions are very useful to measure time or frequency with very high accuracy that the definition of a second is based on them.

Starting with a system of N non-interacting atoms in the ground state |0>, an electromagnetic pulse is applied to create equal superposition of |0> and of an excited state |1> for each atom.

A subsequent free evolution of the atoms for a time t introduces a phase factor between the two states, *w*t, where *w* is the frequency of the transition between |0> and |1>.

At the end of the free evolution, a second electromagnetic pulse is applied and then the probability for the final state in |0> (Ramsey interferometry) is measured.

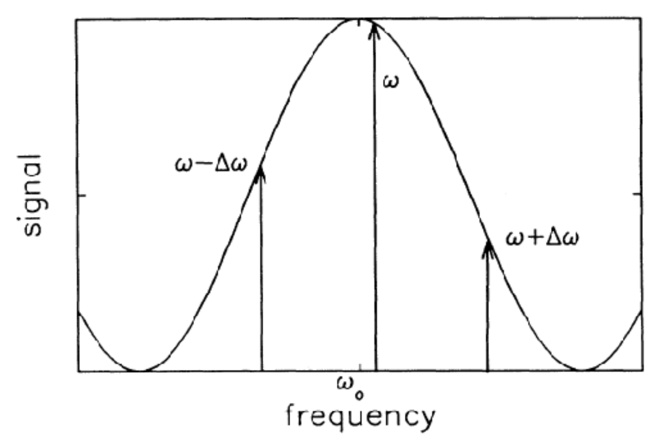
|2>

probe laser

|1>

coupling laser

|0>



(2) High-precision measurements of physical constants

Gravimeters (gravity), gryroscopes (rotation), and gradiometers

$$\phi = (\vec{G} \cdot \vec{g})\tau^2 + 2\vec{G} \cdot (\vec{\Omega} \times \vec{v})\tau^2,$$

Newton's constant G

$$\frac{\phi_{\mathrm{atom}}}{\phi_{\mathrm{light}}} = \frac{mc^2}{\hbar\omega} = \frac{\lambda_{\mathrm{ph}}}{\lambda_{\mathrm{dB}}}\frac{c}{v} \approx 10^{10}.$$

Tests of relativity

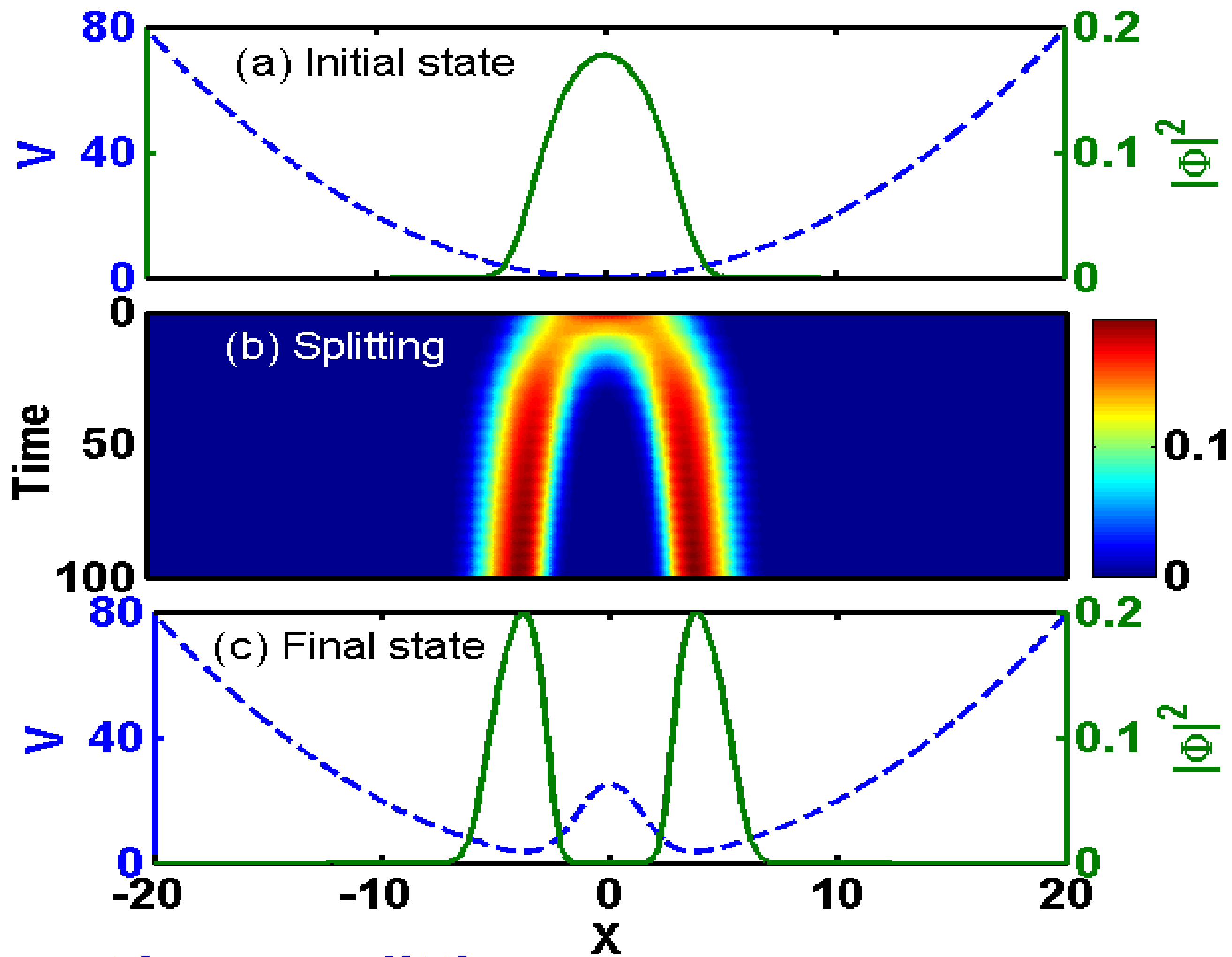
Interferometers in orbit (GPS)

Fine structure constant and ħ/M

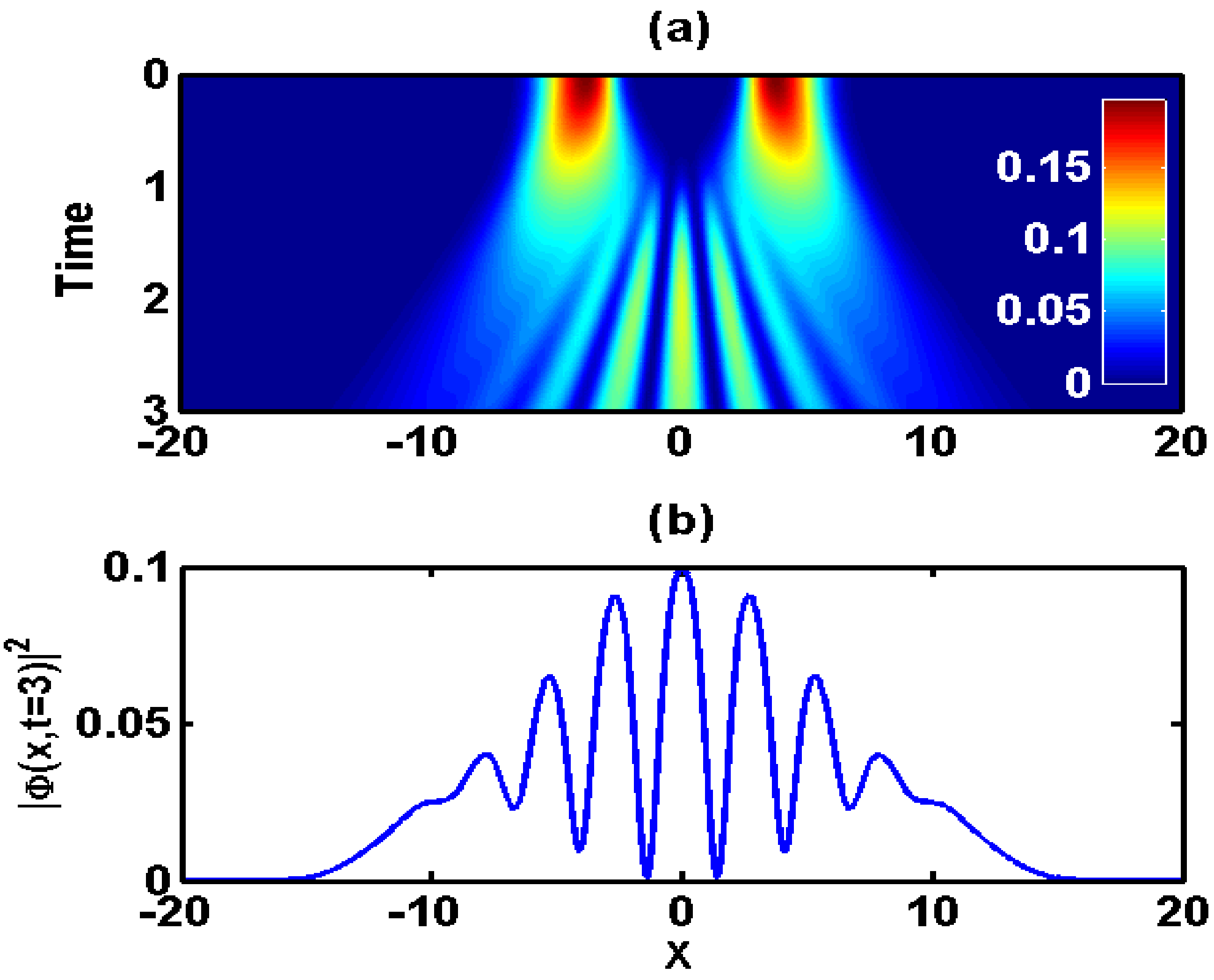
Cronin, Schmiedmayer, Pritchard, Rev. Mod. Phys. 81, 1051 (2009)

2. Matter-wave interferometry

2.1. Atomic matter-wave interference

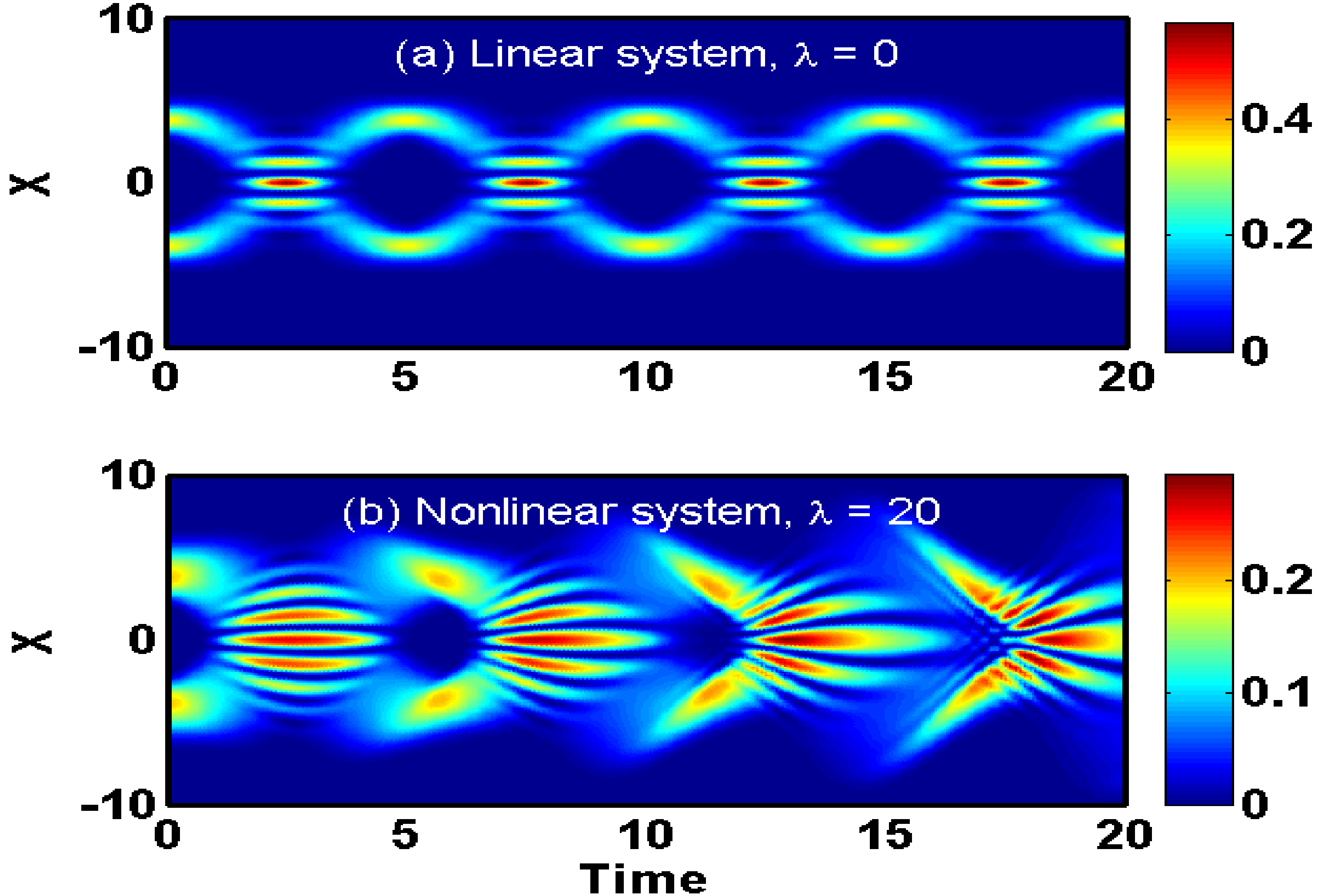


Coherent beam splitting



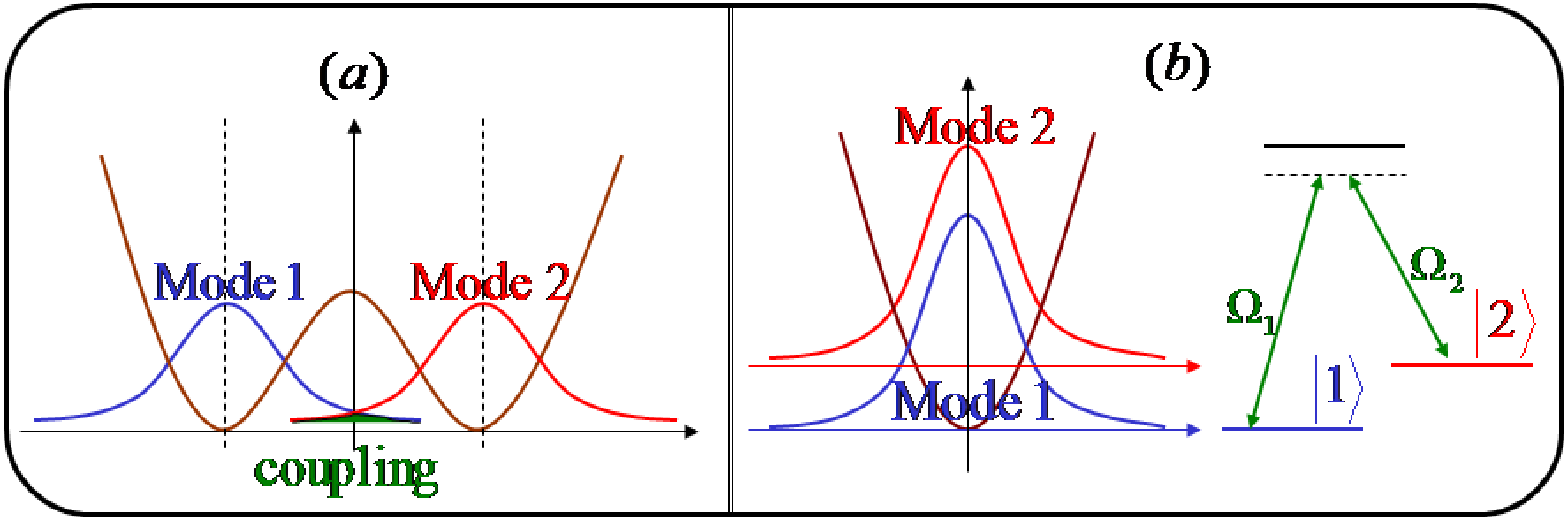
Interference of two freely expanding condensates

2.2. Nonlinear excitations



Nonlinear excitations in 1D matter-wave interference

2.3. Bose-Josephson junction (BJJ)



Schematic diagrams for Bose-Josephson junctions:

(a) an external Bose-Josephson junction linked by quantum tunneling, and

(b) an internal Bose-Josephson junction via a two-component BEC linked by Raman fields.

Unified MF model for both external and internal BJJs

$$H = \frac{\delta}{2}(n_2 - n_1) + \frac{E_c}{8}(n_2 - n_1)^2 - J(\psi_1^*\psi_2 + \psi_2^*\psi_1),$$

with $n_j = \psi_j^*\psi_j = |\psi_j|^2$,

$$\delta = \varepsilon_2 - \varepsilon_1 + N(U_{22} - U_{11})/4,$$

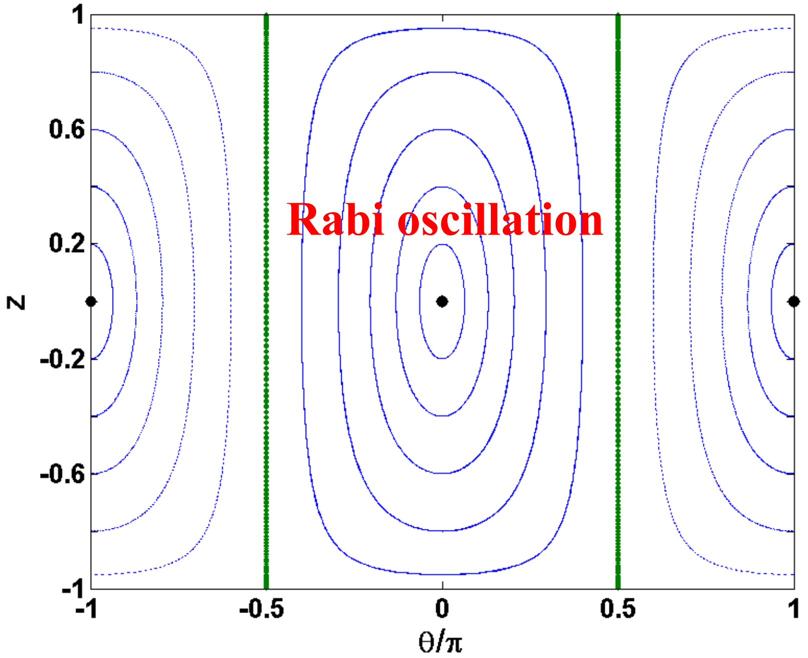
$$E_c = U_{11} + U_{22} \text{ for external BJJs}$$

$$E_c = U_{11} + U_{22} - 2U_{12} \text{ for internal ones.}$$

$$i\hbar\frac{d\psi_1}{dt} = -\frac{\delta}{2}\psi_1 + \frac{E_c}{4}\left(|\psi_1|^2 - |\psi_2|^2\right)\psi_1 - J\psi_2,$$

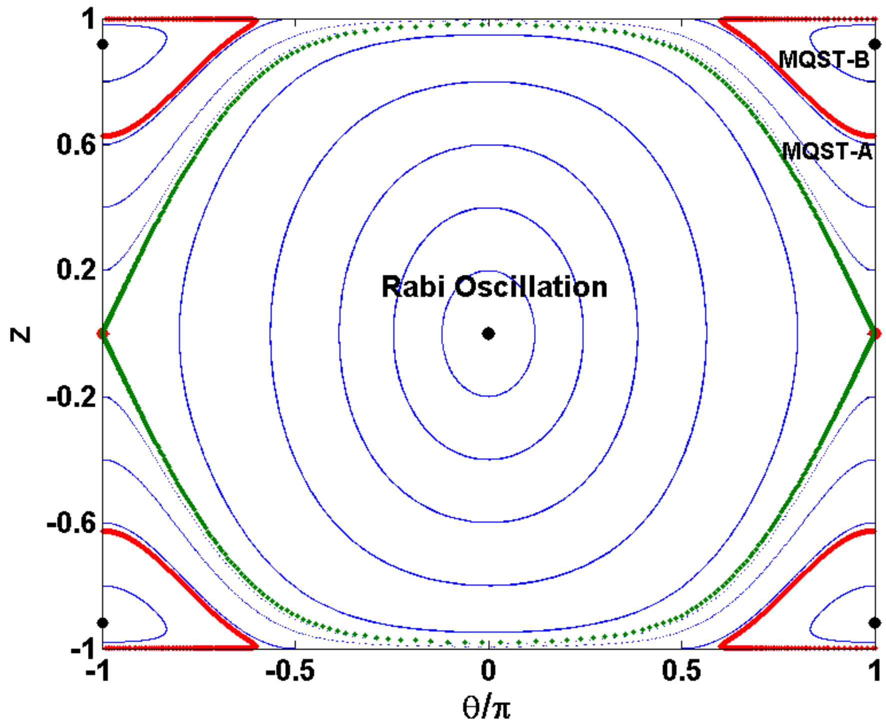
$$i\hbar\frac{d\psi_2}{dt} = +\frac{\delta}{2}\psi_2 + \frac{E_c}{4}\left(|\psi_2|^2 - |\psi_1|^2\right)\psi_2 - J\psi_1.$$

Rabi oscillation and macroscopic quantum self-trapping (MQST)

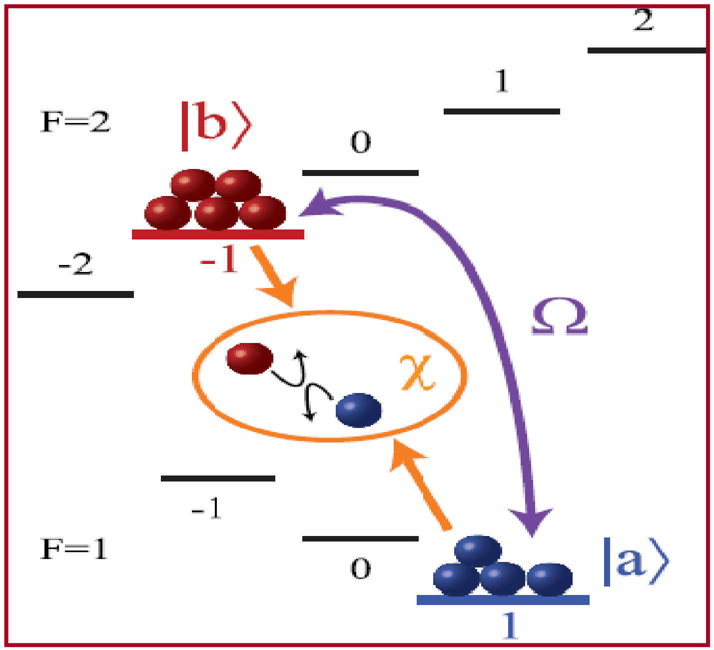


Linear systems, Ec=0

Nonlinear systems, Ec≠0



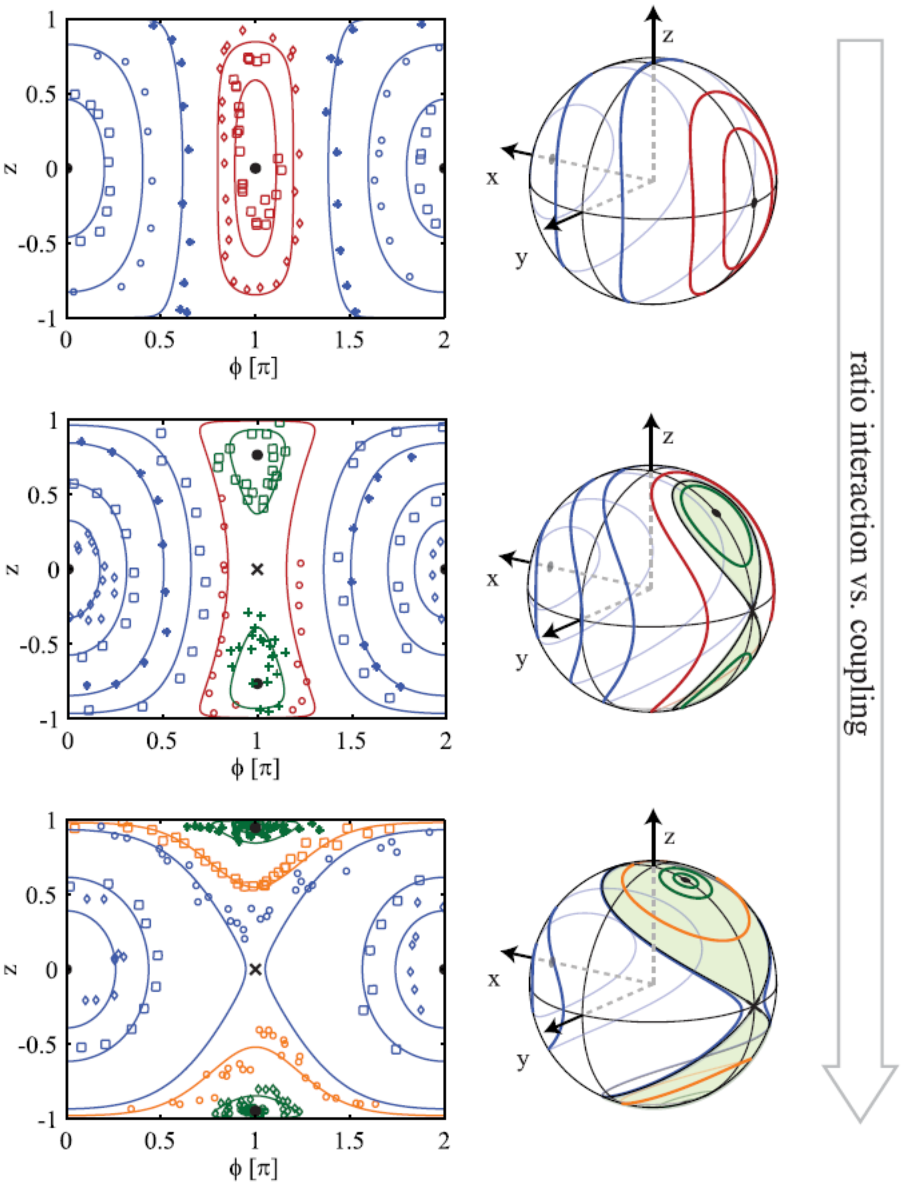
Experimental observation of MQST



$$\hat{H} = \chi \hat{J}_z^2 - \Omega \hat{J}_x, \qquad z = \frac{n_1 - n_2}{n_1 + n_2}$$

classical non - rigid pendulum

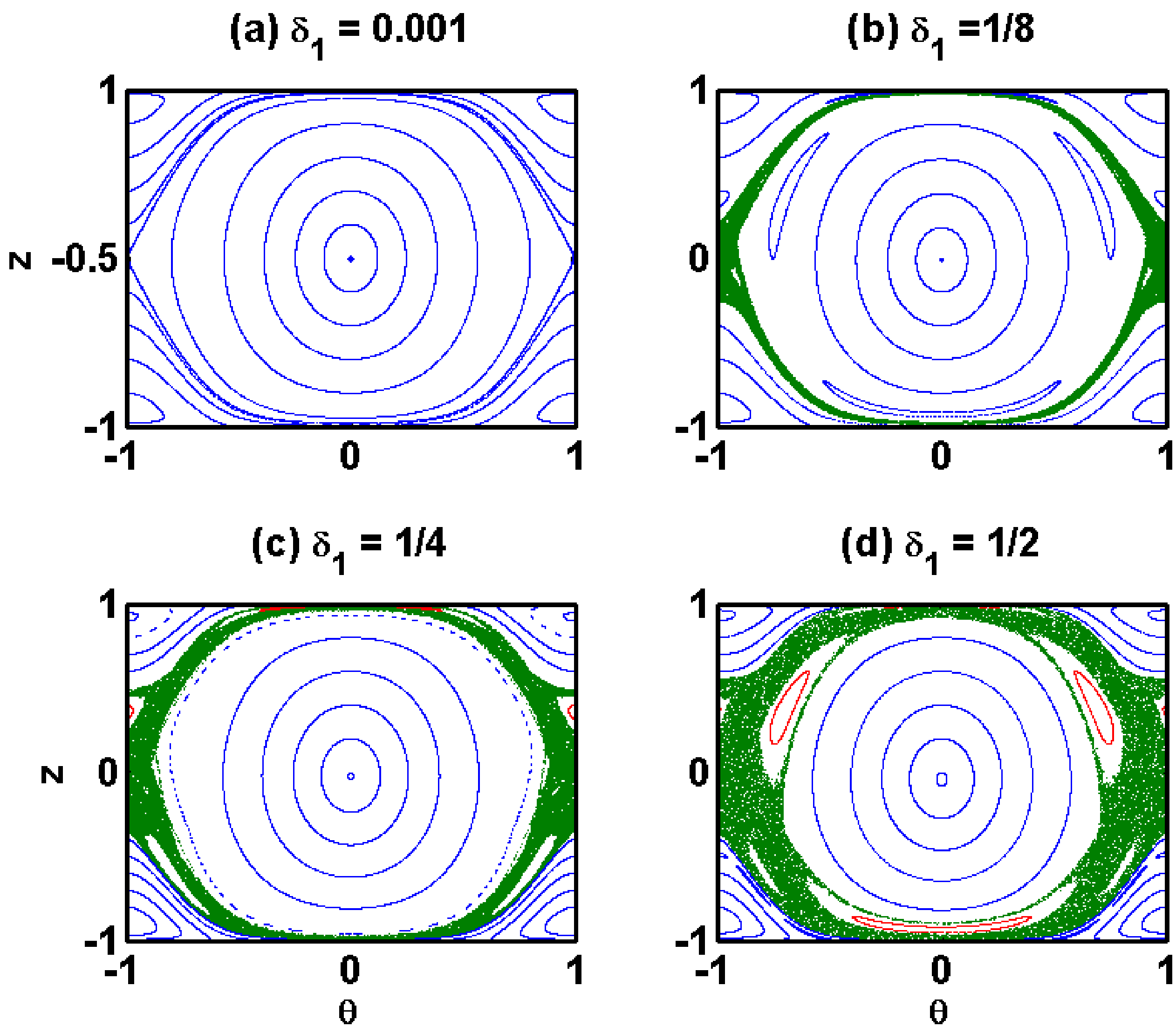
$$H = \chi m^2 - \Omega \sqrt{\left(\frac{N}{2}\right)^2 - m^2} \cos(\phi)$$



Theory:
Smerzi et al, PRL 79, 4950 (1997)

Experiment: Oberthaler et al., PRL 95,010402 (2005); PRL 105, 204101 (2010)

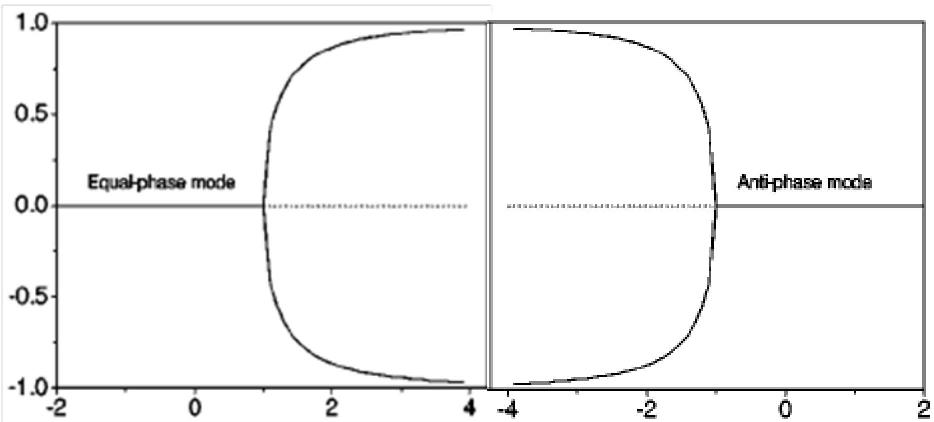
Shapiro resonance and chaos

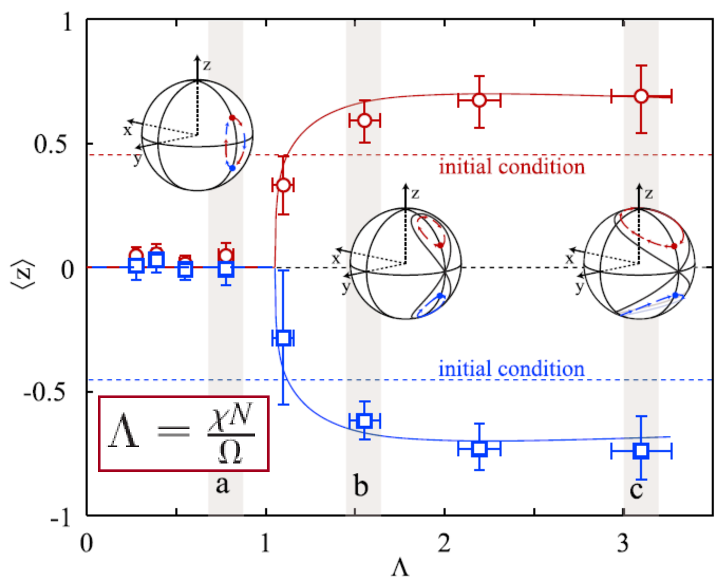


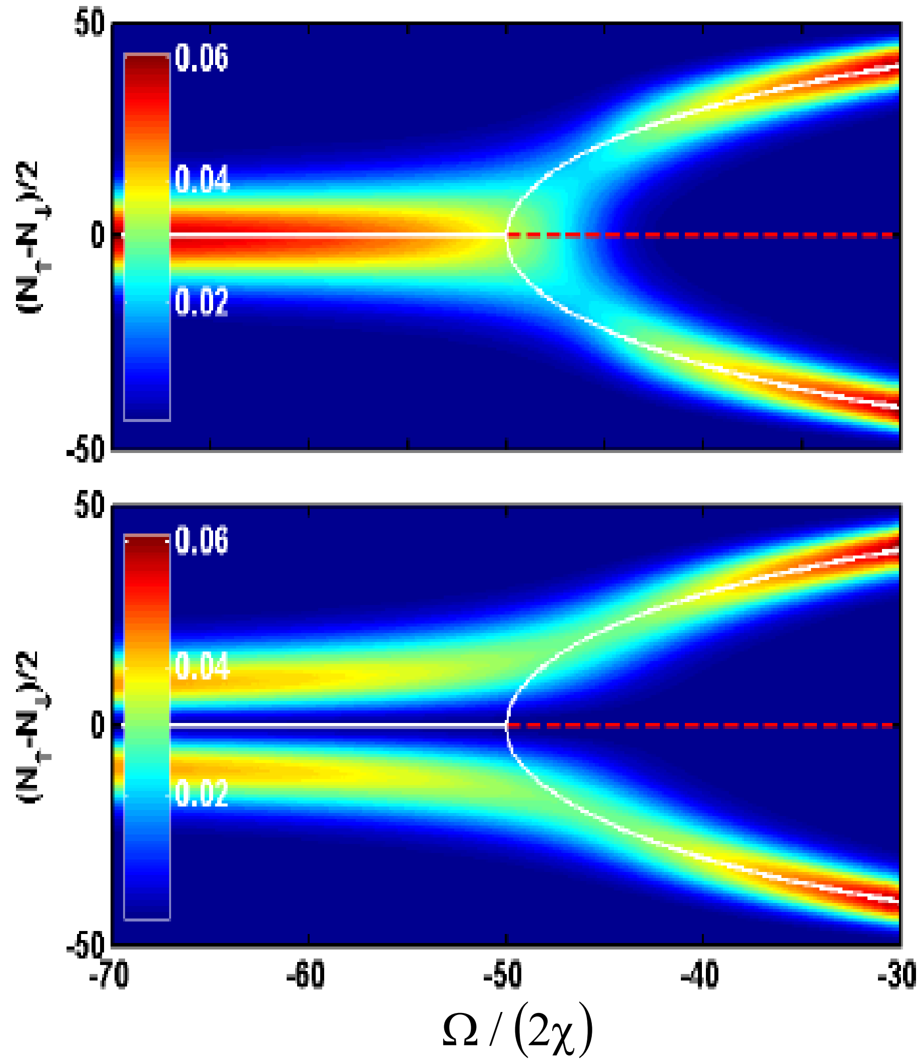
Poincare sections for a BJJ with a driving

$$\delta(t) = \delta_1 \cos(2\pi t).$$

Symmetry-breaking transition







Theory: Lee et al., PRA 69, 033611 (2004); Lee, PRL 102, 070401 (2009); etc.
Experiment: Oberthaler et al., PRL 105, 204101 (2010).

Universal dynamics near critical point

Two characteristic time scales for slow dynsmics across the crtical point,

(1) reaction time (how fast the system follows its ground state),

$$\tau_{\mathrm{r}} = \hbar / \Delta_g(t)$$

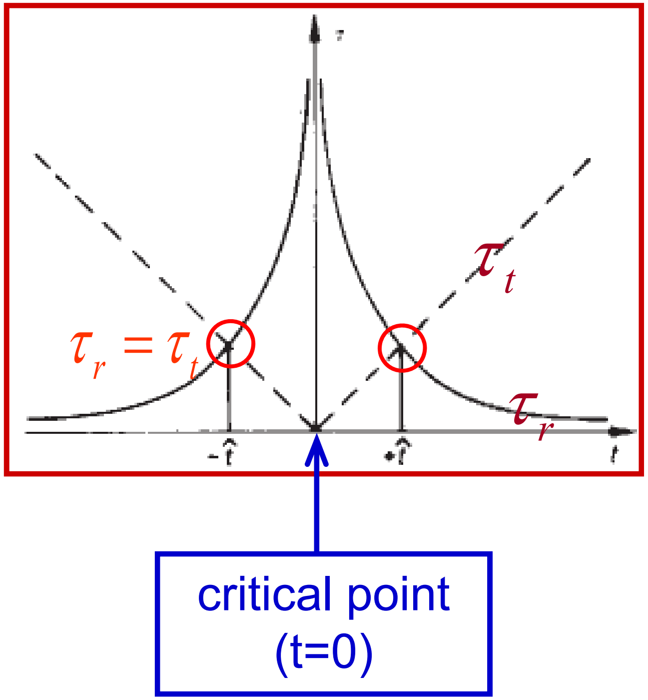
(2) transition time (how fast the system is driven),

$$\tau_{\mathrm{t}} = \Delta_g(t) / \left| \frac{d\Delta_g(t)}{dt} \right|$$

The excitation gap over the ground state

$$\Delta_g(t) = \begin{cases} \sqrt{\hbar\Omega(\hbar\Omega + E_C L)} & \text{for } |\hbar\Omega / E_C| \geq \mathrm{L} \\ \sqrt{(E_C L)^2 - (\hbar\Omega)^2} & \text{for } |\hbar\Omega / E_C| \leq \mathrm{L} \end{cases}$$

where, $\mathrm{L} = \mathrm{N}/2,\ E_C = 2\chi \propto (g_{11} + g_{22} - 2g_{12})$



$\tau_{\mathrm{r}} < \tau_{\mathrm{t}}$, adiabatic evolution

$\tau_{\mathrm{r}} > \tau_{\mathrm{t}}$, non - adiabatic evolution

Kibble-Zurek scalings near critical point

$$\tau_r(\hat{t}) = \tau_t(\hat{t})$$

slow transitions, $\tau_q \gg 1$

$$|\hat{t}| \sim \tau_0^{2/3} \tau_q^{1/3}$$

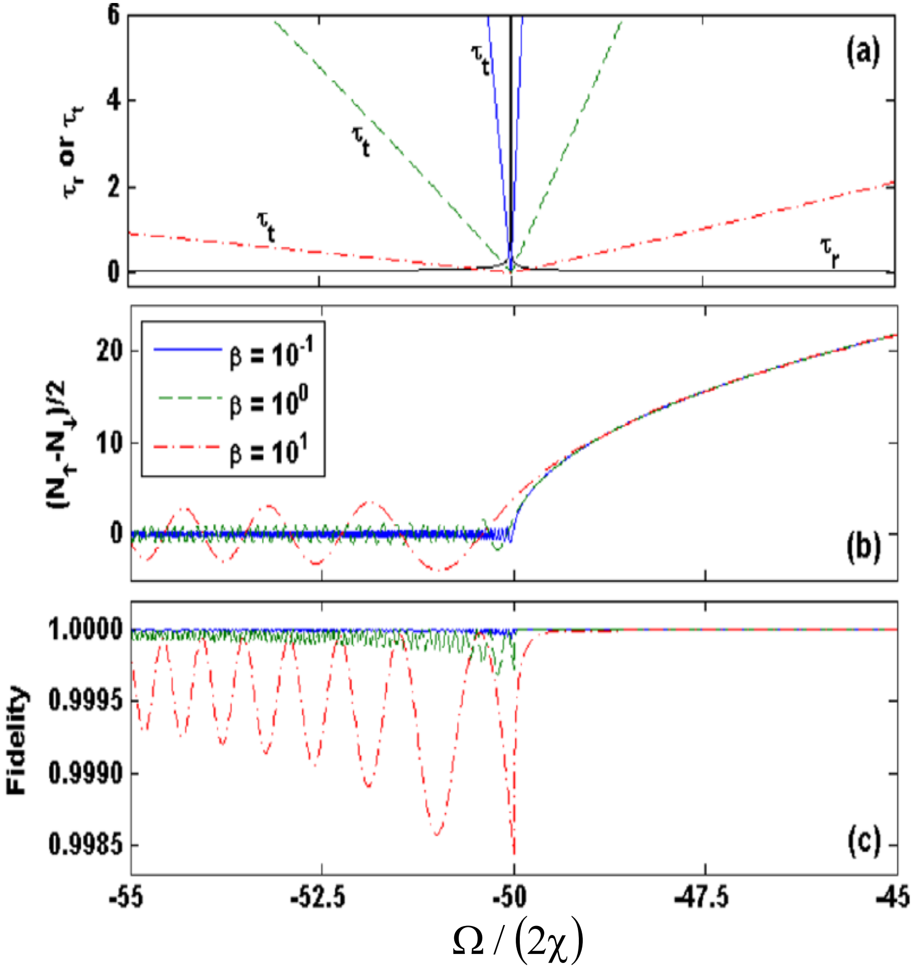
$$\varepsilon \sim \tau_0^{-2/3} \tau_q^{-2/3}$$

Kibble-Zurek scalings

$$|\hat{t}| \sim \tau_0^{1/(1+z\nu)} \tau_q^{z\nu/(1+z\nu)}$$

$$\varepsilon \sim \tau_0^{-1/(1+z\nu)} \tau_q^{-1/(1+z\nu)}$$

with $z = 1$ and $\nu = 1/2$



$$\Omega(t) = \Omega_c(1 \pm t/\tau_q) = \Omega_c \pm \beta t$$

$$\varepsilon = |[\Omega(t) - \Omega_c]/\Omega_c| = |t|/\tau_q$$

$$\tau_0 = 1/\Omega_c$$

Lee, PRL 102, 070401 (2009)

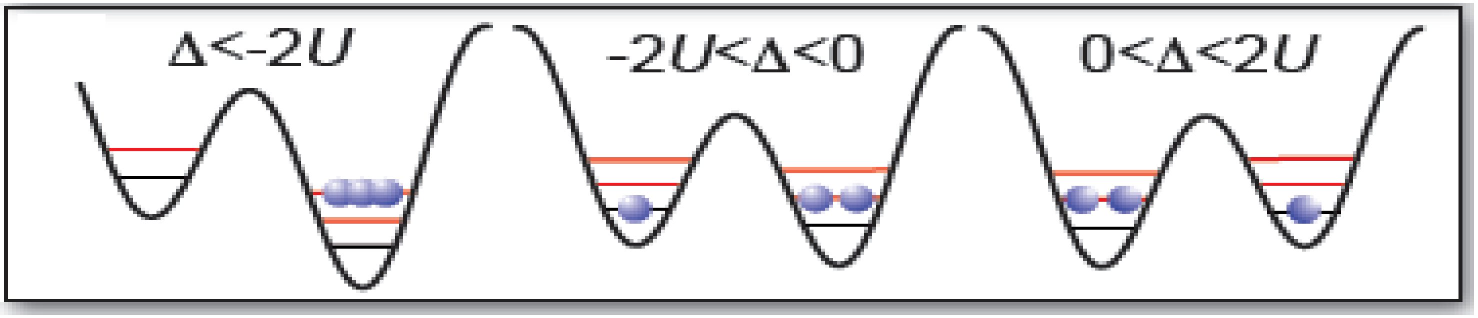
3. Many-body quantum interferometry

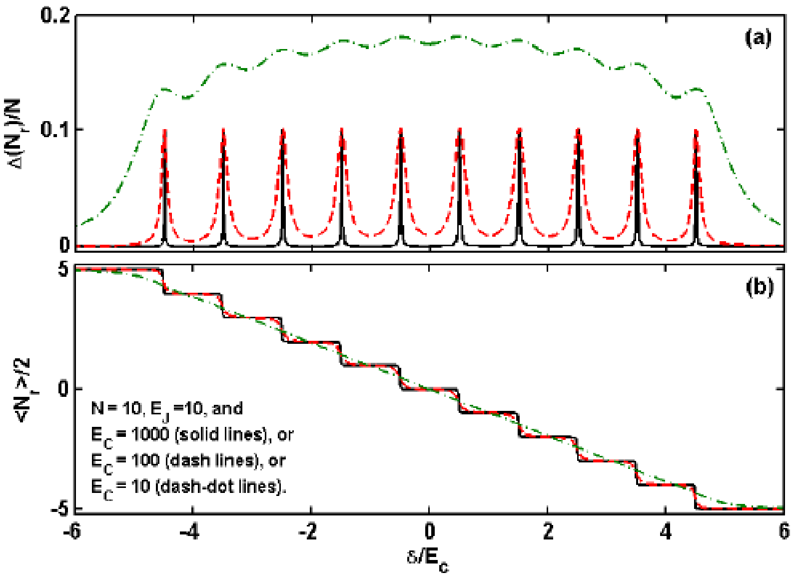
$$H/\hbar = -\frac{\Omega}{2}(a_2^+ a_1 + a_1^+ a_2) + \frac{E_C}{8}(n_2 - n_1)^2 + \frac{\delta}{2}(n_2 - n_1) = -\vec{B}\cdot\vec{J} + \chi J_z^2$$

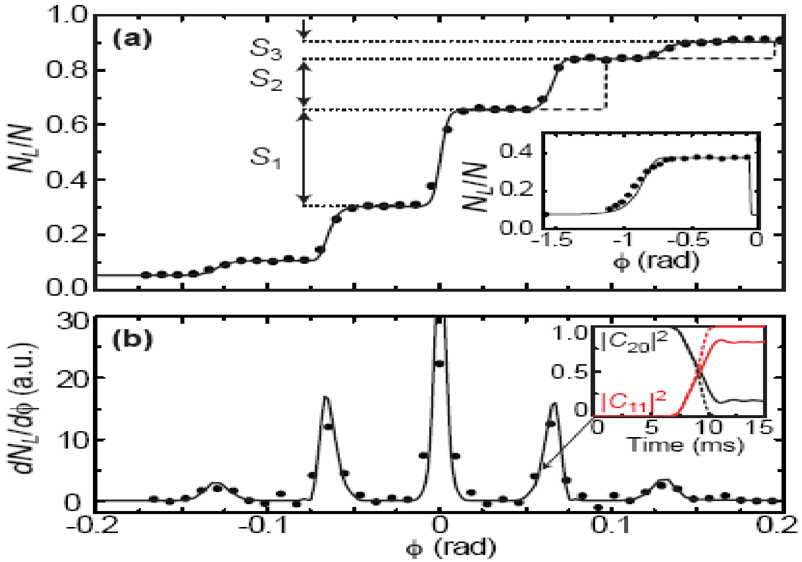
Ground states for symmetric systems, $H/\hbar = -\Omega J_x + \chi J_z^2$

Regime	$\lvert\chi/\Omega\rvert >> 1$ $\chi > 0$	$\lvert\chi/\Omega\rvert \approx 0$	$\lvert\chi/\Omega\rvert >> 1$ $\chi < 0$
State form	$\frac{(a_1^+)^{N/2}(a_2^+)^{N/2}\lvert 0\rangle}{(N/2)!}$	$\frac{(a_1^+ + a_2^+)^N \lvert 0\rangle}{2^{N/2}\sqrt{N!}}$	$\frac{((a_1^+)^N + (a_2^+)^N)\lvert 0\rangle}{2^{1/2}\sqrt{N!}}$
Coherent matrix $\langle a_i^+ a_j\rangle$	$\frac{N}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\frac{N}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$\frac{N}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$
Fluctuations	$\Delta N_i \sim 0$	$\Delta N_i \sim \sqrt{N}$	$\Delta N_i \sim N$

Resonant tunneling and interaction blockade in asymmetric systems







Theory

C. Lee, L.-B. Fu, and Yu. S. Kivshar, EPL 81, 60006 (2008); Carr et al., …

Experiment

P. Cheinet, I. Bloch, et al., Phys. Rev. Lett. 101, 090404 (2008)

3.1. Quantum spin squeezing and many-particle entanglement

Quantum spin squeezing

Squeezing parameter based on the Heisenberg uncertainty relation

$[J_\alpha, J_\beta] = i\varepsilon_{\alpha\beta\gamma} J_\gamma$, $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol.

The uncertainty relation is $(\Delta J_\alpha)^2 (\Delta J_\beta)^2 \geq |\langle J_\gamma \rangle|^2 / 4$.

$$\xi_H^2 = \frac{2(\Delta J_\alpha)^2}{|\langle J_\gamma \rangle|}, \; \alpha \neq \gamma \in (x, y, z), \text{ squeezing parameter}$$

if $\xi_H^2 < 1$, the state is squeezed.

Squeezing parameter ξ_S^2 *given by Kitagawa and Ueda*

$$\xi_S^2 = \frac{\min\left(\Delta J_{\vec{n}_\perp}^2\right)}{j/2} = \frac{4 \min\left(\Delta J_{\vec{n}_\perp}^2\right)}{N},$$

$\vec{n}_\perp$ refers to an axis perpendicular to the MSD

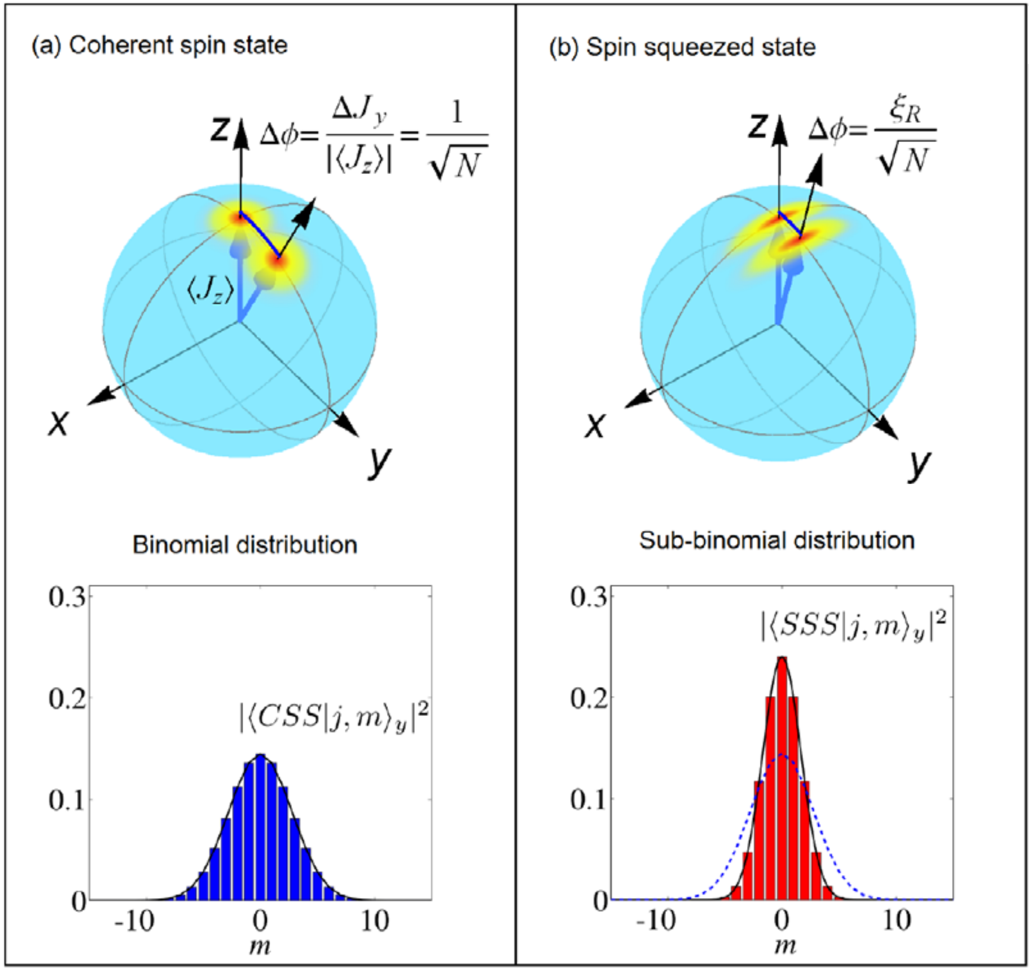
the mean-spin direction (MSD) $\vec{n}_0 = \dfrac{\langle \vec{J} \rangle}{|\langle \vec{J} \rangle|}$

the minimization is over all directions

Jian Ma, Xiaoguang Wang, C. P. Sun, and Franco Nori, arXiv:1011.2978

Squeezing parameter ξ_R^2 given by Wineland et al.

$$\xi_R^2 = \left(\frac{\Delta\phi}{(\Delta\phi)_{\mathrm{CSS}}} \right)^2 = \frac{N\left(\Delta J_{\vec{n}_\perp}\right)^2}{\left|\langle \vec{J} \rangle\right|^2}$$



rotate the state around the x-axis.

$$J_y^{\mathrm{out}} = \exp(i\phi J_x) J_y \exp(-i\phi J_x)$$
$$= \cos\phi J_y - \sin\phi J_z$$

the phase sensitivity $\Delta\phi$

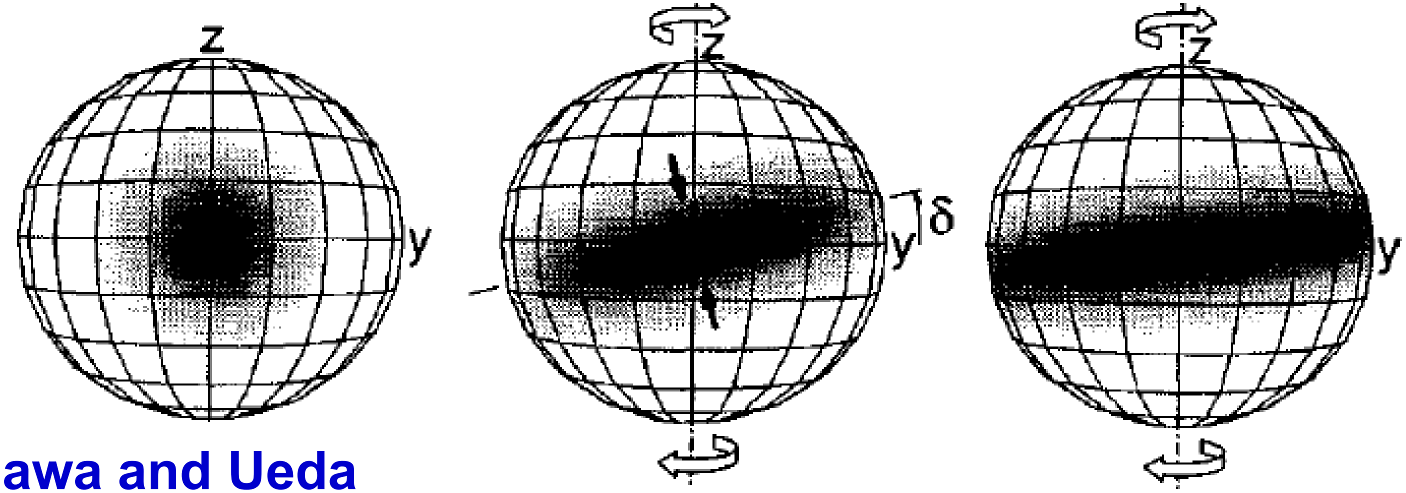
$$\Delta\phi = \frac{\Delta J_y^{\mathrm{out}}}{\left|\frac{\partial \langle J_y^{\mathrm{out}} \rangle}{\partial \phi}\right|} = \frac{\Delta J_y^{\mathrm{out}}}{|\cos\phi \langle J_z \rangle|}$$

standard quantum limit (SQL)

$$(\Delta\phi)_{\mathrm{CSS}} = \frac{\sqrt{j/2}}{j} = \frac{1}{\sqrt{2j}} = \frac{1}{\sqrt{N}},$$

$$\xi_R^2 = \left(\frac{j}{|\langle \vec{J} \rangle|} \right)^2 \xi_S^2$$

Preparing spin squeezing by nonlinear interactions



Kitagawa and Ueda

(a) (b) (c)

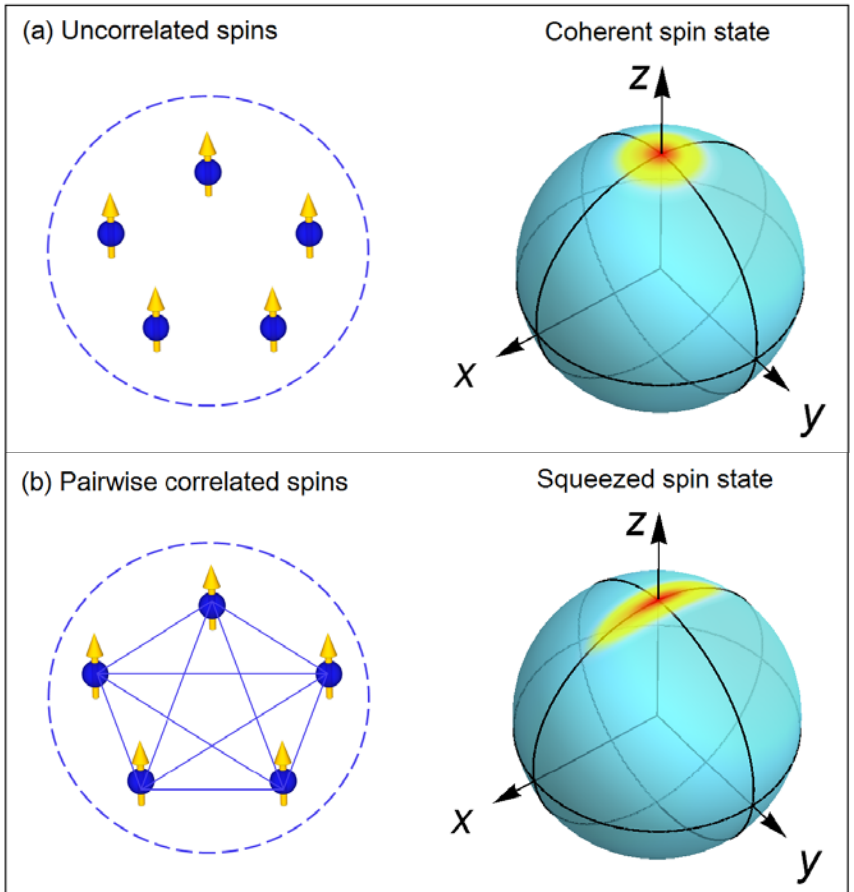
One-axis twisting can reduce the noise down to the order of $S^{1/3}$

FIG. 2. State evolutions by one-axis twisting in terms of the quasiprobability distribution (QPD) on the sphere for $S = 20$. The densities of the figures are normalized by the maximum value $Q_{\max}$ of $Q(\theta, \phi)$. (a) shows the initial coherent spin state $|\theta=\frac{\pi}{2}, \phi=0\rangle$ ($Q_{\max} = 1$). (b) and (c) show one-axis twisted states generated by the unitary transformation $U = \exp[-i\mu S_z^2/2]$; (b) optimally squeezed at $\mu = 0.199$ ($Q_{\max} = 0.445$) and (c) excessively twisted at $\mu = 0.399$ ($Q_{\max} = 0.241$). Although not clear from the figure, the QPD of (c) deviates from a geodesic (swirliness).

Spin squeezing and entanglement

A symmetric state is entangled if and only if it violates the inequality,

$$1 - \frac{4\langle J_{\vec{n}}\rangle^2}{N^2} \geq \frac{4(\Delta J_{\vec{n}})^2}{N} \quad \Leftrightarrow \quad \xi_D^2 = \frac{N(\Delta J_{\vec{n}})^2}{N^2/4 - \langle J_{\vec{n}}\rangle^2} \equiv \frac{N(\Delta J_{n_1})^2}{\langle J_{n_2}\rangle^2 + \langle J_{n_3}\rangle^2} \geq 1$$



Many-particle entanglement with Bose–Einstein condensates

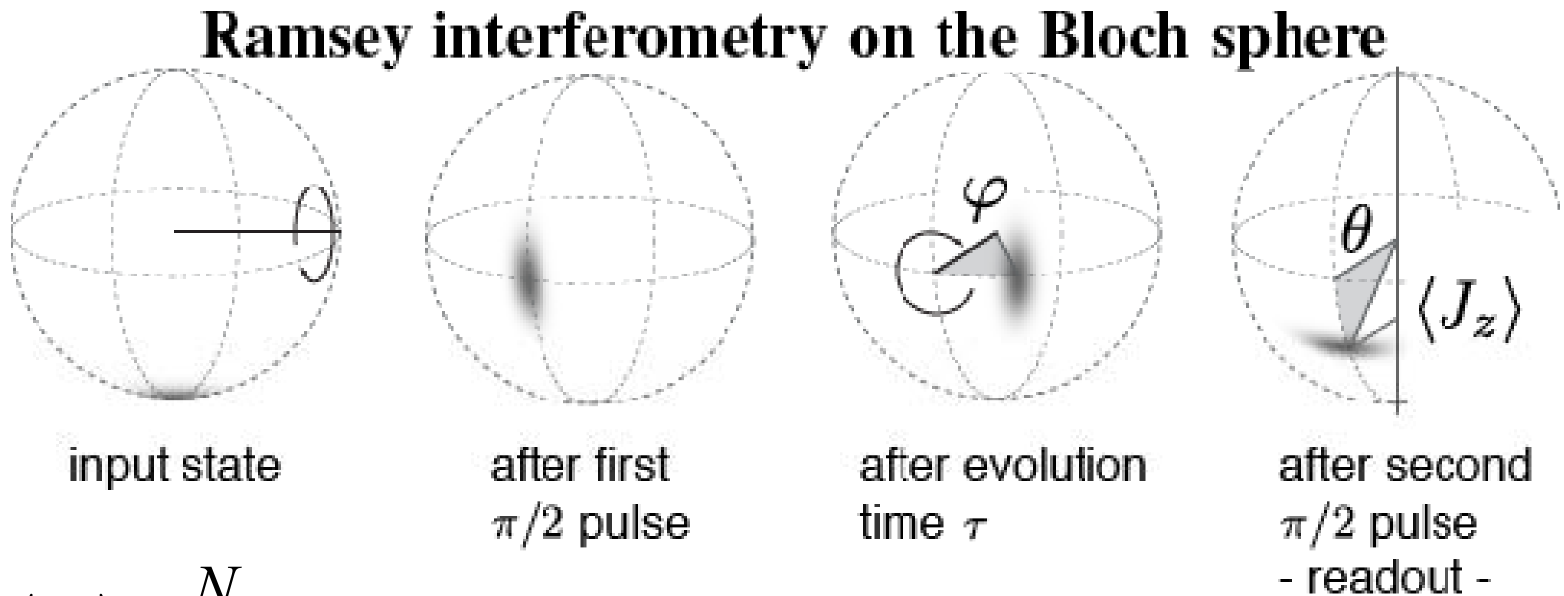
A. Sørensen*, L.-M. Duan†, J. I. Cirac† & P. Zoller†

** Institute of Physics and Astronomy, University of Aarhus, DK-8000 Århus C, Denmark*
† Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria

NATURE | VOL 409 | 4 JANUARY 2001 | www.nature.com

S. Raghavan, H. Pu, P. Meystre, and N. Bigelow,
Generation of arbitrary Dicke states in spinor Bose-Einstein condensates,
Opt. Commun. **188**, 149 (2001)

3.2. High-precision interferometry via spin squeezed states



$$\langle J_z \rangle = \frac{N}{2}\cos\phi,$$

$$\left(\partial \langle J_z \rangle / \partial \phi\right)_{\max} = \frac{N}{2},$$

$$\Delta(J_z) = \frac{\sqrt{N}}{2}\xi_R,$$

$$\Delta(\phi) = \frac{\Delta(J_z)}{\partial \langle J_z \rangle / \partial \phi} = \frac{\xi_R}{\sqrt{N}} \rightarrow \text{phase sensitivity}$$

$\xi_R = 1$, spin coherent state

$\xi_R < 1$, spin squeezed state

Dependent on ξ_R, $\Delta(\phi)$ achieves from standard quantum limit, Heisenberg limit, to super - Heisenberg limit.

Fast diabatic spin squeezing by one axis twisting evolution

$H/\hbar = \chi J_z^2 + \Omega J_\gamma + \Delta\omega_0 J_z$, where $J_\gamma = J_x \cos\gamma + J_y \sin\gamma$ (Kitagawa, Ueda)

nature Vol 464 | 22 April 2010 | doi:10.1038/nature08988

LETTERS

strong nonlinearity via controlling spatial overlap

Atom-chip-based generation of entanglement for quantum metrology

Max F. Riedel[1,2], Pascal Böhi[1,2], Yun Li[3,4], Theodor W. Hänsch[1,2], Alice Sinatra[3] & Philipp Treutlein[1,2,5]

Vol 464 | 22 April 2010 | doi:10.1038/nature08919 nature

LETTERS

strong nonlinearity via using Feshbach resonance

Nonlinear atom interferometer surpasses classical precision limit

C. Gross[1], T. Zibold[1], E. Nicklas[1], J. Estève[1]† & M. K. Oberthaler[1]

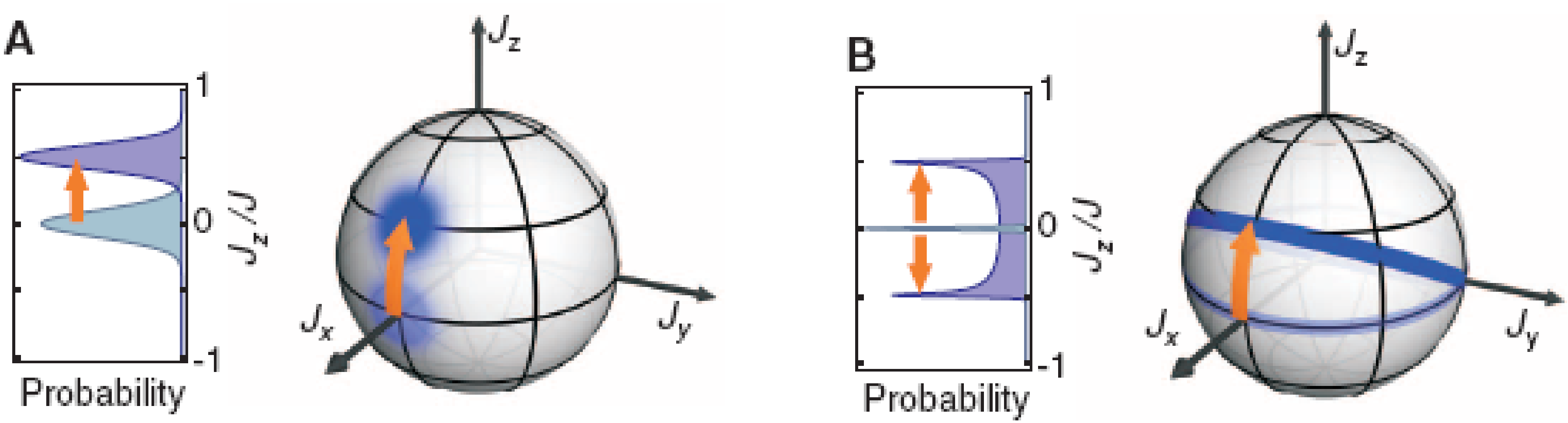
Twin Matter Waves for Interferometry Beyond the Classical Limit

B. Lücke, *et al.*

Science **334**, 773 (2011);
DOI: 10.1126/science.1208798

pair-correlated states from spin dynamics

Interferometers with atomic ensembles are an integral part of modern precision metrology. However, these interferometers are fundamentally restricted by the shot noise limit, which can only be overcome by creating quantum entanglement among the atoms. We used spin dynamics in Bose-Einstein condensates to create large ensembles of up to 10^4 pair-correlated atoms with an interferometric sensitivity $-1.61^{+0.98}_{-1.1}$ decibels beyond the shot noise limit. Our proof-of-principle results point the way toward a new generation of atom interferometers.



3.3. High-precision interferometry via NOON states

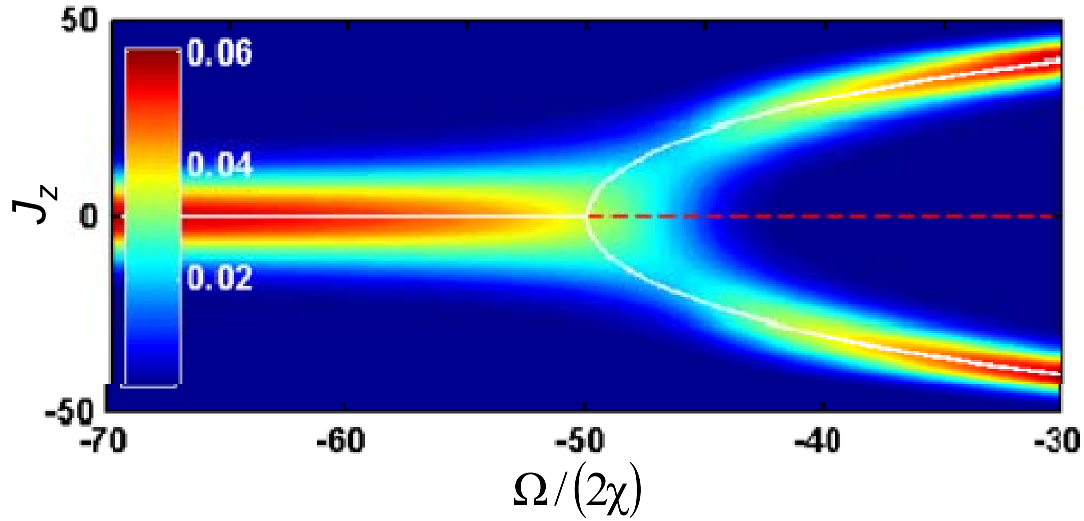
$$H/\hbar = \frac{\delta}{2}(n_1 - n_2) - \frac{\Omega}{2}(a_2^+ a_1 + a_1^+ a_2) + \frac{E_C}{8}(n_1 - n_2)^2 = \delta J_z - \Omega J_x + \chi J_z^2$$

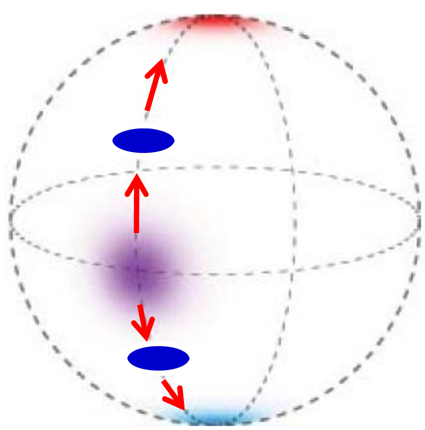
$$\text{Fock basis}: |\text{NOON}\rangle = \frac{1}{\sqrt{2}}\left(|n_1 = N, n_2 = 0\rangle + |n_1 = 0, n_2 = N\rangle\right)$$

$$\text{spin basis}: |\text{NOON}\rangle = \frac{1}{\sqrt{2}}\left(\left|J = \frac{N}{2}, J_z = -\frac{N}{2}\right\rangle + \left|J = \frac{N}{2}, J_z = +\frac{N}{2}\right\rangle\right)$$

The NOON state is a ground state for system of $\delta = 0$, $\chi < 0$ and $|\Omega/\chi| << 1$

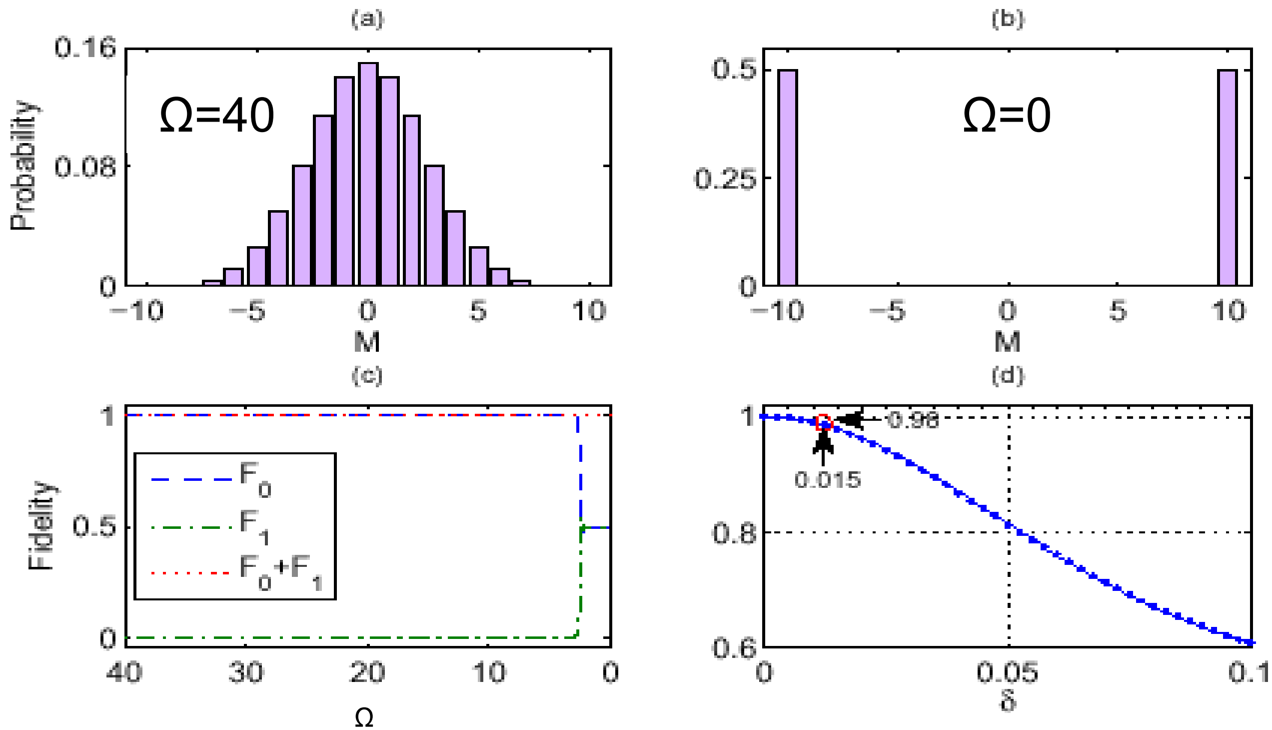
Adiabatic preparation of NOON state via dynamical bifurcation





C. Lee, PRL 97, 150402 (2006)

Beam splitting and recombination via dynamical bifurcation



For a system of $\delta = 0$ and $\chi < 0$, if $\Omega = 40 \rightarrow \Omega = 0$,

$|\mathrm{GS}\rangle = |\mathrm{CS}\rangle_{\mathrm{SU(2)}} \rightarrow |\mathrm{NOON}\rangle = \left(|\mathrm{P1}\rangle + |\mathrm{P2}\rangle\right)/\sqrt{2}$.

Here, $|\mathrm{P1}\rangle = |\mathrm{J} = \mathrm{N}/2, \mathrm{M} = -\mathrm{N}/2\rangle$ and $|\mathrm{P2}\rangle = |\mathrm{J} = \mathrm{N}/2, \mathrm{M} = +\mathrm{N}/2\rangle$ are the ground and first - excited states for the system of $\Omega = 0$ and $0 < \delta < |\chi|$, respectively. They can be used as two paths of a MZ interferometer.

Phase accumulation via the term of δJ_z

Switch on the term δJ_z for a period of time T,

$$|\mathrm{NOON}\rangle \to \frac{1}{\sqrt{2}}\left(e^{-i\delta T\cdot(N/2)}|\mathrm{P1}\rangle + e^{+i\delta T\cdot(N/2)}|\mathrm{P2}\rangle\right)$$

with $\varphi = \delta T$, which is the phase accumulated in a single-atom system.

Extract the relative phase from the population information via a dynamical bifurcation from $|\Omega/\chi|<<1$ to $|\Omega/\chi|>>1$

Due to the indistinguishability, we can not use the proposals of Wineland et al. and Caves et al.

At the side of $|\Omega/\chi| << 1$, the ground [first excited] states will be $\left(|\mathrm{P1}\rangle + |\mathrm{P2}\rangle/\sqrt{2}\right)$ $[\left(|\mathrm{P1}\rangle - |\mathrm{P2}\rangle\right)/\sqrt{2}]$ even for a very small Ω.

Therefore, the state after the dynamical bifurcation becomes

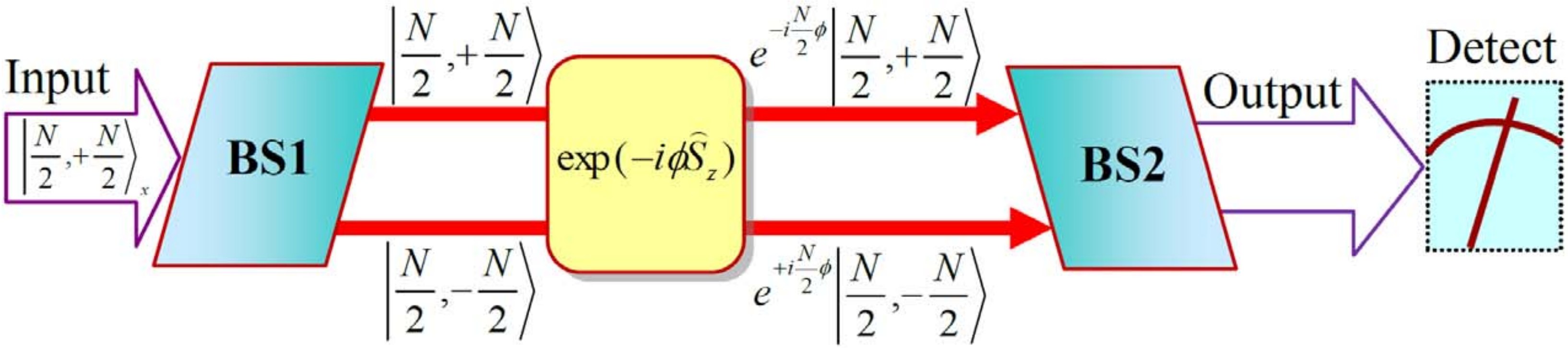
$$\cos(N\varphi/2)|\mathrm{GS}\rangle - i\cdot\sin(N\varphi/2)|\mathrm{FS}\rangle,$$

whose populations are $P_{GS} = \cos^2(N\varphi/2) = (1+\cos(N\varphi))/2$

and $P_{FS} = \sin^2(N\varphi/2) = (1-\cos(N\varphi))/2.$

Schematic diagram for MZ interferometry via NOON states of indistinguishable systems

Hamiltonian, $H/\hbar = \delta J_z - \Omega J_x + \chi J_z^2$



State Evolution

$$\left|\frac{N}{2},-\frac{N}{2}\right\rangle \Rightarrow |CS\rangle_{SU(2)} \Rightarrow \frac{|P1\rangle + |P2\rangle}{\sqrt{2}} \Rightarrow \frac{e^{-\frac{i}{2}N\varphi}|P1\rangle + e^{+\frac{i}{2}N\varphi}|P2\rangle}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\frac{N\varphi}{2}\right)|GS\rangle - i\cdot\sin\left(\frac{N\varphi}{2}\right)|FS\rangle \Rightarrow \cos\left(\frac{N\varphi}{2}\right)|P1\rangle - i\cdot\sin\left(\frac{N\varphi}{2}\right)|P2\rangle$$

with

$$|P1\rangle = \left|\frac{N}{2},-\frac{N}{2}\right\rangle \text{ and } |P2\rangle = \left|\frac{N}{2},+\frac{N}{2}\right\rangle$$

Keynotes

- negative nonlinearity ($\chi < 0$) $\rightarrow$ Feshbach resonance
- coupling $\rightarrow$ tunnelling (double-well system), or
  Raman transition (two-component condensate)
- two paths $\rightarrow$ two degenerated ground states for the system of $\chi < 0$
- beam splitting/recombination $\rightarrow$ dynamical bifurcation
- path entangled state (NOON state) $\rightarrow$ dynamical bifurcation

Advantages

- large total number of particle (in order of 10^3, 10 for systems of photons and trapped ions)
- reduced influence of environment (adiabatic evolution and closed sub-Hilbert space)
- measurement precision of Heisenberg limit (path entangled states)
- experimental possibility (double-well or two-component systems)

Challenge

- adiabatic evolution requests long coherent time

C. Lee, PRL 97, 150402 (2006)

4. Summary and open problems

Summary

- In interferometers of Bose condensed atoms, the atom-atom interaction brings the nonlinearity to the system.
- Tuning the effective nonlinearity, symmetry-breaking transitions appear and the dynamics near the critical point obey the universal Kibble-Zurek mechanism.
- The spin squeezed states and NOON state can be prepared by controlling the nonlinearity and these states can used for high-precision interferometry beyond the standard quantum limit.

Open Problems

- noises (quantum fluctuations and technical noises)
- imperfect effects (atom loss and environment)
- coupling between internal and external degrees of freedom
- finite-temperature effects

More details in, C. Lee, et al., arXiv:1110.4734v3 (a review article)

Thanks for your attention!

International senior scientist (1000-talent program, our university) and postdoctoral positions available (my group)!